7. The curve C has equation

$$y = \frac{k^2}{x} + 1 \qquad x \in \mathbb{R},\ x \neq 0$$

where k is a constant.

(a) Sketch C stating the equation of the horizontal asymptote. **(3)**

The line l has equation $y = -2x + 5$

(b) Show that the x coordinate of any point of intersection of l with C is given by a solution of the equation

$$2x^2 - 4x + k^2 = 0$$

(2)

(c) Hence find the exact values of k for which l is a tangent to C.

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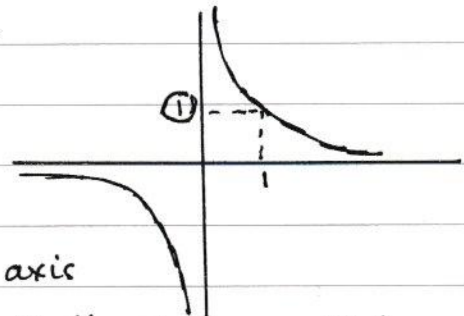
(3)

$y = \frac{1}{x}$ looks like

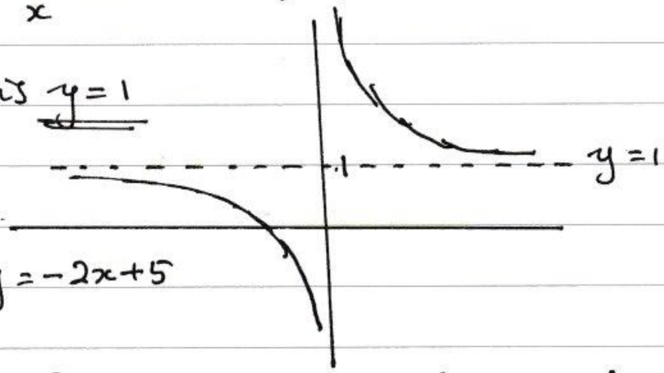


$y = \frac{k^2}{x}$ would be similar

but the numbers on the y axis would be k^2 larger. The 1 in the circle would become k^2

If we add 1, so $y = \frac{k^2}{x} + 1$ the graph is shifted up by 1

The horizontal asymptote is $\underline{y = 1}$



Solving $y = \frac{k^2}{x} + 1$ and $y = -2x + 5$

Gives:

$$\frac{k^2}{x} + 1 = -2x + 5 \Rightarrow \frac{k^2}{x} = -2x + 4 \Rightarrow \underline{2x^2 - 4x + k^2 = 0}$$

If l is a tangent there is only one solution to this equation so the discriminant $b^2 - 4ac = 0$

$\Rightarrow 16 - 8k^2 = 0$

$\Rightarrow k^2 = 2$

$\Rightarrow \underline{k = \pm\sqrt{2}}$