10. A circle C has equation

$$x^2 + y^2 - 4x + 8y - 8 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C.

(3)

The straight line with equation $x = k$, where k is a constant, is a tangent to C.

(b) Find the possible values for k.

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(2)

(a) (i) & (ii) Need to express the equation as

$$(x-a)^2 + (y-b)^2 - r^2 = 0$$

(a) (i)(ii)

$$x^2 + y^2 - 4x + 8y - 8 = 0$$

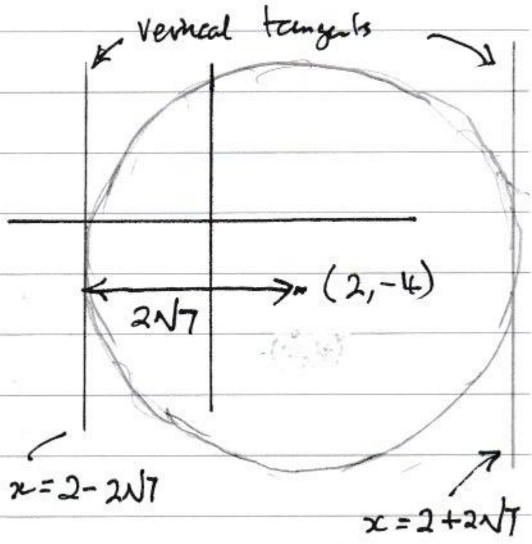
$$(x-2)^2 - 4 + (y+4)^2 - 16 = 8$$

$$(x-2)^2 + (y+4)^2 = 28$$

So centre is at $\underline{(2,-4)}$ radius $= \sqrt{28} = \underline{2\sqrt{7}}$

(b) This is easiest to see if you draw a diagram and note $x = k$ is a line parallel to y axis,

So values of k are $\underline{2 \pm 2\sqrt{7}}$



You can also do this algebraically.

If $x = k$ is a tangent

then the solution of $x = k$ and

$x^2 + y^2 - 4x + 8y - 8 = 0$ has a discriminant of zero.

Substituting k for x

$$\underbrace{y^2}_{a=1} + \underbrace{8y}_{b} + \underbrace{(k^2 - 4k - 8)}_{c} = 0$$

and then plough on with $(b^2 - 4ac) = 0$

It gives the same answer but this is just two marks, so I think the geometrical answer above is preferable