14. The circle C has equation

$$x^2 + y^2 - 6x + 10y + 9 = 0$$

(a) Find

(i) the coordinates of the centre of C

(ii) the radius of C

(3)

The line with equation $y = kx$, where k is a constant, cuts C at two distinct points.

(b) Find the range of values for k.

2018

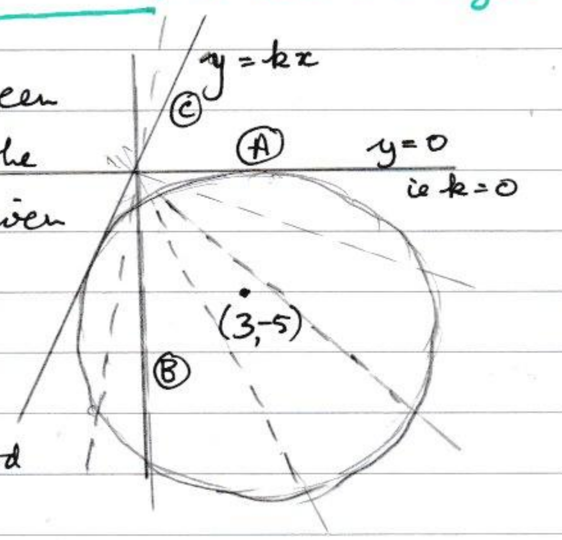
(6)

Usual start!

(a) $(x-3)^2 - 9 + (y+5)^2 - 25 + 9 = 0$

$(x-3)^2 + (y+5)^2 = 25$

(i) (ii) So centre is at $(3,-5)$ radius $= 5$ $\Rightarrow$ x axis is a tangent



(b) Here is a picture. All the lines between $y=0$ and the one marked $y = kx$ cut the circle in two points. All the lines between $y = 0$ (A) and $x = 0$ (B) have k negative. Between $x = 0$ (B) and $y = kx$ (C) k has a +ve steep value. This should all come out of the algebra but it is good to see the picture.

picture it.

Putting $y = kx$

$x^2 + (kx)^2 - 6x + 10(kx) + 9 = 0$

$\Rightarrow$ $x^2(1+k^2) + x(10k - 6) + 9 = 0$

For two solutions for x the discriminant > 0 $(b^2 - 4ac > 0)$

$(10k-6)^2 - 36(1+k^2) > 0$

$64k^2 - 120k = 0$ at the boundaries for value of k

Gives $k = 0$ (ie $y = 0$ the x axis as seen above)

$k = 120/64 = 15/8$. (corresponds to line C)

So between A an B $k < 0$

between B ad C $k > 15/8$

So either k is $-$ve or it must be greater than $15/8$

The algebra makes sense with the diagram.