11. (i) A circle C_1 has equation

$$x^2 + y^2 + 18x - 2y + 30 = 0$$

The line l is the tangent to C_1 at the point $P(-5, 7)$.

Find an equation of l in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(5)

(ii) A different circle C_2 has equation

$$x^2 + y^2 - 8x + 12y + k = 0$$

where k is a constant.

Given that C_2 lies entirely in the 4th quadrant, find the range of possible values for k. 2020

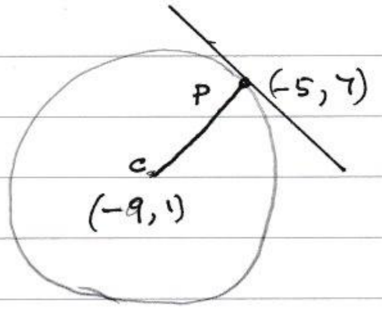
(4)

(i) If l is a tangent it is $\perp^r$ to the radius at that point. To find the gradient of the tangent we need to find the gradient of the radius line from the centre to P. So we need to find the coordinates of the centre of the circle.

As in example 1 we write C_1 as

$$(x+9)^2 - 81 + (y-1)^2 - 1 + 30 = 0$$

So the centre is at $(-9, 1)$



The gradient of CP is $\dfrac{7-1}{-5-(-9)} = \dfrac{6}{4} = \dfrac{3}{2}$

So the gradient of the tangent is $\perp^r$

ie l has gradient $-2/3$.

So l is of the form $y = -\frac{2}{3}x + k$

But l passes through $(-5, 7)$ so $7 = -\frac{2}{3}(-5) + k$ (k to save confusion with c in question)

$$k = \frac{11}{3}$$

So l has equation $y = -\frac{2}{3}x + \frac{11}{3}$ or $\underline{\underline{3y + 2x - 11 = 0}}$

(ii) Next pdf page.

(ii) $x^2 + y^2 - 8x + 12y + k = 0$

To lie entirely in the fourth quadrant the radius must be less than the shortest distance to one of the axes. So we need to find the centre to start. Note that this does NOT depend on k. k only affects the radius

$$(x-4)^2 - 16 + (y+6)^2 - 36 + k = 0$$

$$\text{So } (x-4)^2 + (y+6)^2 = (52-k)$$

So the centre is at $(4, -6)$ and the radius is $\sqrt{52-k}$

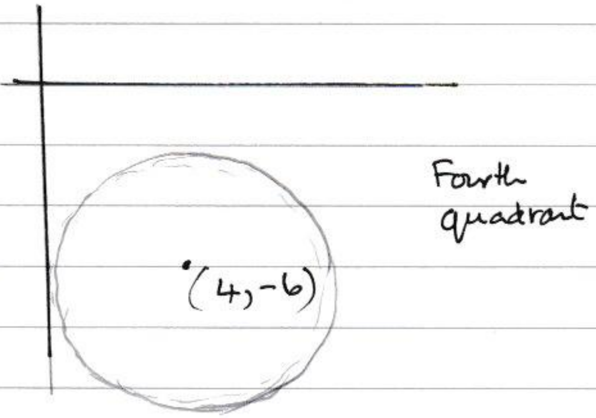
To fit in the fourth quadrant $r < 4$

$$r^2 < 16$$

$$\text{so } 52 - k < 16$$

$$-k < -36$$

$$\underline{k > 36}$$



Also $\underline{r}$ cannot be negative So $52 - k > 0$ otherwise r^2 is negative

So $k < 52$

So range for k is $36 < k < 52$

Don't miss this