15.

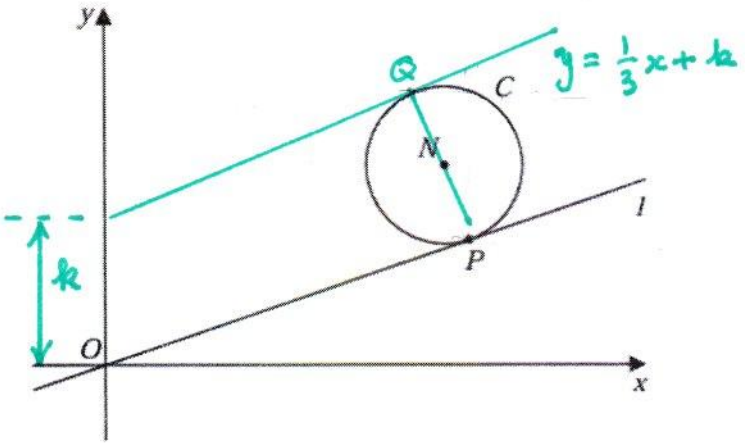

Figure 4

Figure 4 shows a sketch of a circle C with centre $N(7, 4)$

The line l with equation $y = \frac{1}{3}x$ is a tangent to C at the point P.

Find

(a) the equation of line PN in the form $y = mx + c$, where m and c are constants,

(2)

(b) an equation for C.

(4)

The line with equation $y = \frac{1}{3}x + k$, where k is a non-zero constant, is also a tangent to C.

(c) Find the value of k.

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(3)

(a) PN is $\perp^r$ to l so has gradient -3. It also passes through N so

$y = -3x + c$ and $\quad = -21 + c \quad$ so $c = 25$

PN is therefore $\underline{y = -3x + 25}$

(b) We know the centre of the circle. The radius is PN. N is at $\underline{(7, 4)}$ so we want the coordinates of P. This is the intersection of l and PN ie the intersection of

$y = x/3$ (i) and $y = -3x + 25$ (ii)

Putting (i) in (ii) $\quad x/3 = -3x + 25$

$10x/3 = 25$, $\quad x = 7.5 \quad$ so $y = \frac{1}{3}x = 2.5$

So P is $\underline{(7.5, 2.5)}$

So $r^2 = (7.5 - 7)^2 + (4 - 2.5)^2 = 2.5 = 5/2$

So equation of circle is $\underline{(x-7)^2 + (y-4)^2 = 5/2}$

(Next page of pdf for (c))

The other possibility for k is shown by the extra green line on the diagram.

Q has coordinates $(6.5, 5.5)$ as it is an equal projection from $P(7.5, 2.5)$ to $N(7, 4)$

But $y = \frac{x}{3} + k$ must pass through Q

$$5.5 = \frac{6.5}{3} + k$$

$$k = \frac{16.5 - 6.5}{3}$$

$$= \frac{10}{3}$$