12. (a) Show that

$$\frac{10\sin^2\theta - 7\cos\theta + 2}{3 + 2\cos\theta} \equiv 4 - 5\cos\theta$$

(4)

(b) Hence, or otherwise, solve, for $0 \leqslant x < 360°$, the equation

$$\frac{10\sin^2 x - 7\cos x + 2}{3 + 2\cos x} = 4 + 3\sin x$$

2019

(3)

(a) Get all in terms of $\cos\theta$ – use $\sin^2\theta + \cos^2\theta = 1$

$$\text{LHS} = \frac{10(1-\cos^2\theta) - 7\cos\theta + 2}{3+2\cos\theta} = \frac{-10\cos^2\theta - 7\cos\theta + 12}{3+2\cos\theta}$$

If it helps put $x = \cos\theta$ – to make it seem familiar.

$$\frac{-10x^2 - 7x + 12}{3+2x}$$

If this is to equal RHS then numerator must factorise and $3+2x$ must cancel. This is so.

$$= \frac{\cancel{(2x+3)}(-5x+4)}{\cancel{(3+2x)}}$$

$$= 4 - 5x = 4 - 5\cos\theta$$ as required

(b) To solve this use part (a) to simplify LHS. This gives

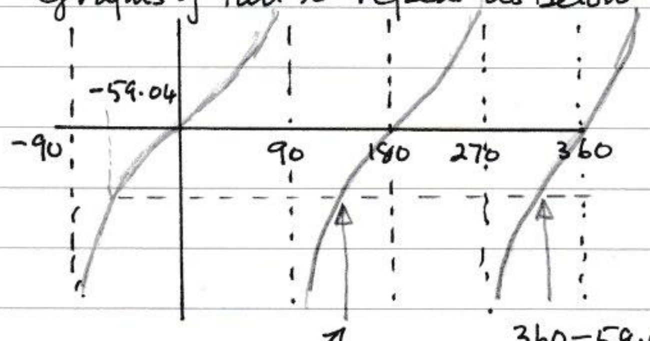
$$4 - 5\cos x = 4 + 3\sin x \Rightarrow -5\cos x = 3\sin x$$

or $\frac{\sin x}{\cos x} = \tan x = -\frac{5}{3}$.

So one solution is $x = \tan^{-1}\left(-\frac{5}{3}\right) = -59.04°$

We are required to find solutions in range $0 \leqslant x < 360°$.

Graphs of $\tan x$ repeat as below



$180 - 59.04$

$\underline{\underline{= 120.96}}$

$360 - 59.04$

$\underline{\underline{= 300.96}}$

Answers required.