12. (a) Show that the equation

$$4\cos\theta - 1 = 2\sin\theta\tan\theta$$

can be written in the form

$$6\cos^2\theta - \cos\theta - 2 = 0$$

(4)

(b) Hence solve, for $0 \leqslant x < 90°$

$$4\cos 3x - 1 = 2\sin 3x\tan 3x$$

giving your answers, where appropriate, to one decimal place.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*) 2018

(4)

(a) Here we need to use $\tan\theta = \sin\theta / \cos\theta$ first

$$4\cos\theta - 1 = 2\sin\theta . \frac{\sin\theta}{\cos\theta}$$

Multiply by $\cos\theta$ $\quad \cos\theta(4\cos\theta - 1) = 2\sin^2\theta$

Now use $\sin^2\theta + \cos^2\theta = 1$ to get all in terms of $\cos\theta$

$$4\cos^2\theta - \cos\theta = 2(1 - \cos^2\theta)$$
$$= 2 - 2\cos^2\theta$$

Rearranging $\underline{6\cos^2\theta - \cos\theta - 2 = 0}$

(b) The equation to solve is in the form of part (a) but $\theta = 3x$.

So we need to solve – using part (a)

$$6\cos^2(3x) - \cos(3x) - 2 = 0$$

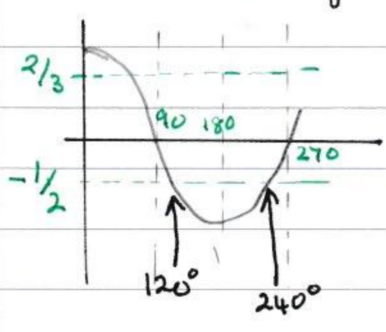
If it helps write $\cos(3x) = y$

$6y^2 - y - 2 = 0$ which factorises

$(3y - 2)(2y + 1) = 0$ giving $y = 2/3$ or $-1/2$.

So we require values of x s.t $\cos(3x) = 2/3$ or $\cos(3x) = -1/2$

in the range $0 - 90°$ for x so in the range $0 \to 270°$ for $3x$



$\cos^{-1}(2/3) = 48.2$ so $x = 1/3(48.2) = \underline{\underline{16.1°}}$

Between 0 and 270° there are two values of $\cos^{-1}(-1/2)$

These are 120° and 240°

so $x = 1/3(120) = \underline{\underline{40°}}$ and $1/3(240) = \underline{\underline{80°}}$

So three values are 16.1°, 40°, and 80°