9.

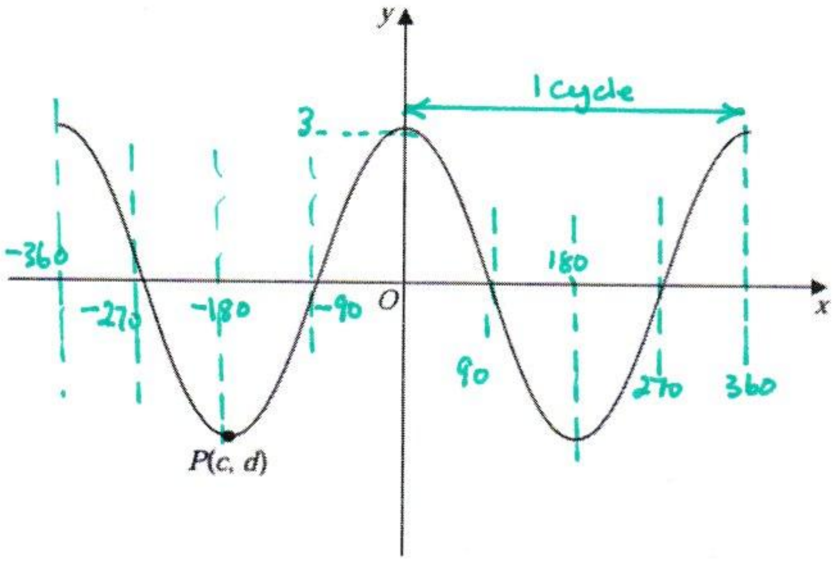


Figure 3

Figure 3 shows part of the curve with equation $y = 3\cos x°$.

The point $P(c, d)$ is a minimum point on the curve with c being the smallest negative value of x at which a minimum occurs.

(a) State the value of c and the value of d.

(1)

(b) State the coordinates of the point to which P is mapped by the transformation which transforms the curve with equation $y = 3\cos x°$ to the curve with equation

(i) $y = 3\cos\left(\frac{x°}{4}\right)$

(ii) $y = 3\cos(x - 36)°$

(2)

(c) Solve, for $450° \leqslant \theta < 720°$,

$$3\cos\theta = 8\tan\theta$$

giving your solution to one decimal place.

In part (c) you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable. 2020

(5)

Maybe an idea to mark on the diagram key values of x as done

(a) Amplitude = 3 so $d = 3$. From the diagram P is at $(-180°, -3)$

(b) (i) When x is divided by 4 x must be 4× bigger for same value

So graph is stretched in the x direction by a factor of 4.

So $P \rightarrow (-720°, -3)$.

(ii) To get same value x must be added to by 36° to get same value

So the graph is translated by 36° to the right

So $P \rightarrow (-180+36, -3)$ ie $(-144°, -3)$

(c) $3\cos\theta = 8\tan\theta$ use $\tan\theta = \sin\theta/\cos\theta$

$= \dfrac{8\sin\theta}{\cos\theta}$

$\Rightarrow \quad 3\cos^2\theta = 8\sin\theta$ Now use $\cos^2\theta + \sin^2\theta = 1$ to get all in terms of $\sin\theta$

$\Rightarrow \quad 3(1-\sin^2\theta) = 8\sin\theta$

rearranging $3\sin^2\theta + 8\sin\theta - 3 = 0$

If you prefer put $\sin\theta = x$, but here we won't do that

This factorises $(3\sin\theta - 1)(\sin\theta + 3) = 0$

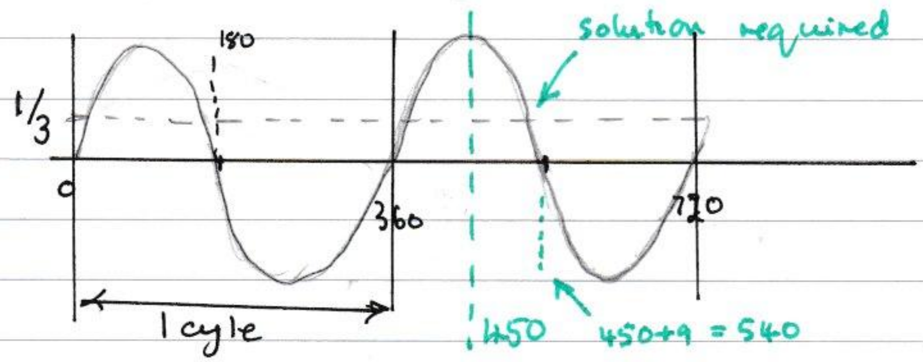
giving $\sin\theta = -3$ ← Max of $\sin\theta = 1$ so this is inadmissible

or $\sin\theta = \frac{1}{3}$

This gives $\theta = 19.5°$.

But we require θ in the range $450° \leq \theta \leq 720$.

Best to draw a repeating sin curve to help.



There are various ways you can look at this. Simplest may be to observe that it is 19.5 less than 540 = $\underline{520.5°}$

(All solutions in range 0 – 720 are $(0+19.5)$, $(180-19.5)$
$(360+19.5)$ $(540-19.5)$ etc for $\underline{>720}$)