

12.

In this question you should show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < \theta \leq 450^\circ$, the equation

$$5 \cos^2 \theta = 6 \sin \theta$$

giving your answers to one decimal place.

(5)

(ii) (a) A student's attempt to solve the question

"Solve, for $-90^\circ < x < 90^\circ$, the equation $3 \tan x - 5 \sin x = 0$ "

is set out below.

$$\begin{aligned} 3 \tan x - 5 \sin x &= 0 \\ 3 \frac{\sin x}{\cos x} - 5 \sin x &= 0 \\ 3 \sin x - 5 \sin x \cos x &= 0 \\ 3 - 5 \cos x &= 0 \\ \cos x &= \frac{3}{5} \\ x &= 53.1^\circ \end{aligned}$$

Identify two errors or omissions made by this student, giving a brief explanation of each.

(2)

The first four positive solutions, in order of size, of the equation

$$\cos(5\alpha + 40^\circ) = \frac{3}{5}$$

are $\alpha_1, \alpha_2, \alpha_3$ and α_4

(b) Find, to the nearest degree, the value of α_4

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(2)

(i)

$$5 \cos^2 \theta = 6 \sin \theta$$

use $\sin^2 \theta + \cos^2 \theta = 1$ to

get all in terms of $\sin \theta$

$$5(1 - \sin^2 \theta) = 6 \sin \theta$$

rearrange

$$5 \sin^2 \theta + 6 \sin \theta - 5 = 0$$

which sadly does not factorise.

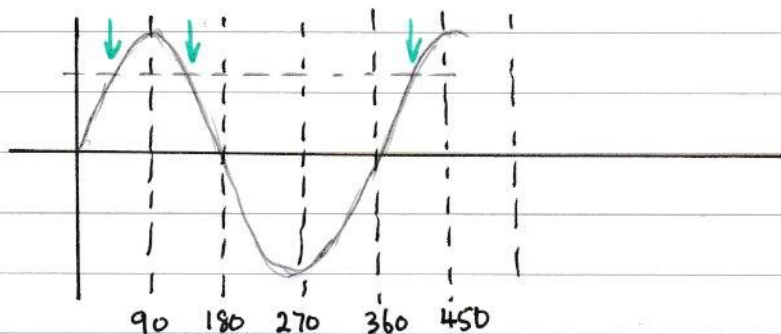
so we will put $\sin \theta = x$ to save writing if you like

$$\sin \theta = \frac{-6 \pm \sqrt{36 + 100}}{10} = \frac{-6 \pm 11.66}{10} = \frac{+5.66}{10}$$

[The -ve sign is inadmissible as this gives $\sin \theta$ as $-17.66/10$ and the smallest value of $\sin \theta$ is -1]

$$\text{So } \theta = \sin^{-1}(+0.566) = +34.47^\circ$$

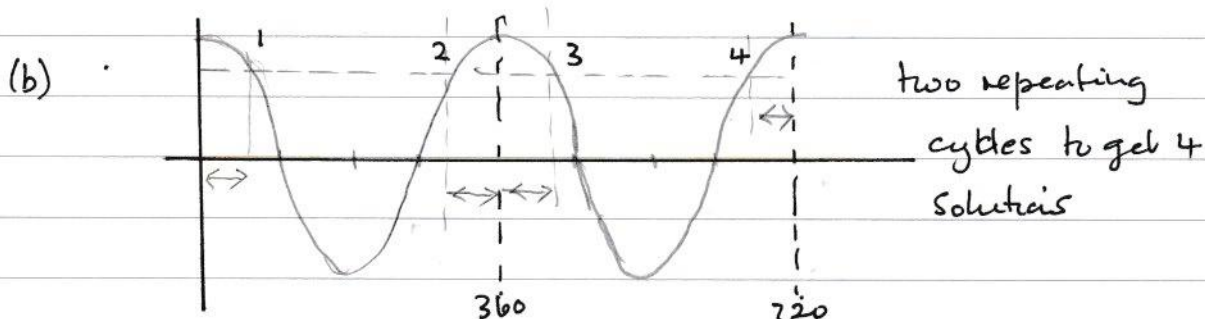
We require solutions in the range $0 \leq \theta < 450^\circ$



From the graph the solutions are 34.5° , $(180^\circ - 34.5^\circ)$, $(360^\circ + 34.5^\circ)$
 ie 34.5° , 145.5° and 394.5°

(ii) (a) • You cannot divide by $\sin x$ (line 3-4) as $\sin x$ being zero is a possible solution - look at the second line. Dividing by zero is not allowed.

• $\cos x$ is symmetrical about the y axis so -53.1° is also a solution in the range.



The 4th solution is $5d_4 + 40 = 720 - 53.1 = 666.9^\circ$

$$\text{so } 5d_4 = 626.9^\circ$$

$$\underline{d_4 = 125.4^\circ} \text{ to 1 dec pl.}$$

[It is possible to use a CAST diagram instead of seeing the cycles as above. The use of CAST may be quicker, but less explanatory - At least to way I think]. Use whichever.