3. (a) Given that k is a constant, find

$$\int \left(\frac{4}{x^3} + kx \right) dx$$

simplifying your answer.

(3)

(b) Hence find the value of k such that

$$\int_{0.5}^{2} \left(\frac{4}{x^3} + kx \right) dx = 8$$

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(3)

$$\int (4x^{-3} + kx)\, dx = \frac{4x^{-2}}{(-2)} + \frac{kx^2}{2} + c$$

$$= \frac{-2}{x^2} + \frac{kx^2}{2} + c$$

$$\left[-\frac{2}{x^2} + \frac{kx^2}{2} \right]_{1/2}^{2} = 8$$

We ignore the c as it will be there when $x = 2$ and when $x = 0.5$ so will cancel.

$$8 = \left(-\frac{2}{4} + \frac{4k}{2} \right) - \left(-\frac{2}{1/4} + \frac{k}{2} \times \frac{1}{4} \right)$$

$$8 = \left(-\frac{1}{2} + 2k \right) - \left(-8 + \frac{k}{8} \right)$$

$$\cancel{8} = -\frac{1}{2} + \cancel{8} + 2k - \frac{k}{8}$$

$$\frac{1}{2} = \frac{16k - k}{8} = \frac{15k}{8}$$

giving $k = \frac{4}{15}$