7. Given that k is a positive constant and $\int_1^k \left(\frac{5}{2\sqrt{x}} + 3\right) dx = 4$

(a) show that $3k + 5\sqrt{k} - 12 = 0$

(4)

(b) Hence, using algebra, find any values of k such that

$$\int_1^k \left(\frac{5}{2\sqrt{x}} + 3\right) dx = 4$$

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(4)

(a) $\int_1^k \left(\frac{5}{2}x^{-1/2} + 3\right) dx = 4 \Rightarrow \left[\frac{5}{2}\frac{x^{1/2}}{(1/2)} + 3x\right]_1^k = 4$

$\left(5k^{1/2} + 3k\right) - (5+3) = 4$

$5\sqrt{k} + 3k - 8 = 4$

$\underline{3k + 5\sqrt{k} - 12 = 0}$ as required.

(b) If the integral is true then we need to find values of k s.t. $3k + 5\sqrt{k} - 12 = 0$ (s.t. = such that)

This is essentially a quadratic if we put $\sqrt{k} = x$

$(3x^2 + 5x - 12) = 0$

This factorises

$(3x - 4)(x + 3) = 0$

Giving $x = \sqrt{k} = \frac{4}{3}$ or -3

This gives $k = 16/9$ or 9 (reject). BUT. k is confined to the +ve domain so when we take $\sqrt{k}$ we would expect to have to take the +ve root. But $\sqrt{k}$ must be -3 for $k = 9$ to work. So we are left with $k = 16/9$ and reject the $k = 9$.

[Note if we took the −ve square root of 16/9 that would also fail but that is in the −ve domain.