10.

$$g(x) = 2x^3 + x^2 - 41x - 70$$

(a) Use the factor theorem to show that $g(x)$ is divisible by $(x - 5)$.

(2)

(b) Hence, showing all your working, write $g(x)$ as a product of three linear factors.

(4)

The finite region R is bounded by the curve with equation $y = g(x)$ and the x-axis, and lies below the x-axis.

(c) Find, using algebraic integration, the exact value of the area of R.

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(4)

This problem involves factor theorem, long division of polynomials, and knowing how to integrate to find area under a curve

(a) When $x = 5$ $g(x) = 2\times125 + 25 - 205 - 70 = 0$ so $(x-5)$ is a factor.

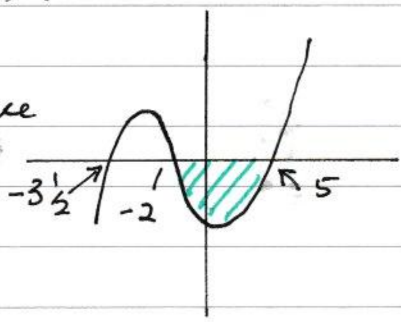
(b) $g(x)$ ← dividing by $(x-5)$

$$\begin{array}{r|l} & 2x^2 + 11x + 14 \\ \hline x-5 & 2x^3 + x^2 - 41x - 70 \\ & \underline{2x^3 - 10x^2} \\ & \qquad 11x^2 - 41x \\ & \qquad \underline{11x^2 - 55x} \\ & \qquad\qquad 14x - 70 \\ & \qquad\qquad \underline{14x - 70} \end{array}$$

But $2x^2 + 11x + 14$ factorises to $(2x+7)(x+2)$

Giving $g(x) = \underline{(x-5)(2x+7)(x+2)}$.

(c) Roots are $5, -3\frac{1}{2}, -2$ The graph looks like



So we require to find the shaded area

$$= \int_{-2}^{5} (2x^3 + x^2 - 41x - 70)\, dx$$

$$= \left[\frac{x^4}{2} + \frac{x^3}{3} - \frac{41x^2}{2} - 70x\right]_{-2}^{5} = \left(\frac{625}{2} + \frac{125}{3} - \frac{1025}{2} - 350\right) - \left(\frac{16}{2} - \frac{8}{3} - 82 + 140\right)$$

$$= -\frac{1525}{3} - \left(\frac{190}{3}\right) = -\frac{1715}{3} = \underline{\underline{-571\tfrac{2}{3}}}.$$

The area is $-ve$ as it is below the axis.