13.

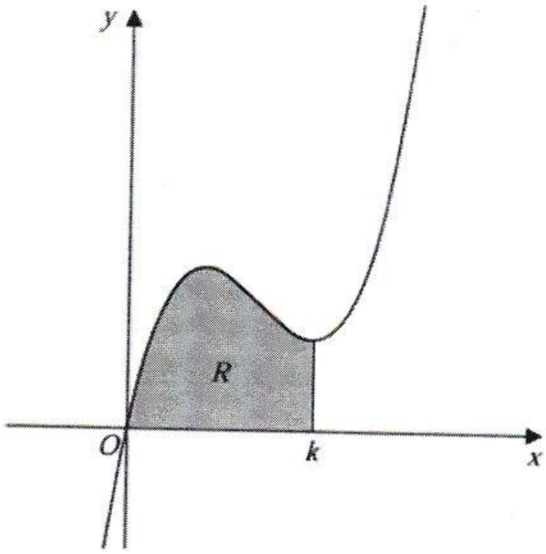

Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 2x^3 - 17x^2 + 40x$$

The curve has a minimum turning point at $x = k$.

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line with equation $x = k$.

Show that the area of R is $\dfrac{256}{3}$

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

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(7)

We need to find the value of k. k is at a minimum so differentiate. $\frac{dy}{dx} = 6x^2 - 34x + 40 = (3x-5)(2x-8) = 0$

This is zero when $x = 5/3$ or $x = 4$. $x = 5/3$ is at the max
$x = 4$ $(> 5/3)$ is at the minimum so $k = 4$.

$$\text{Area } R = \int_0^4 \left(2x^3 - 17x^2 + 40x\right) dx$$

$$= \left[\frac{2x^4}{4} - \frac{17x^3}{3} + \frac{40x^2}{2}\right]_0^4 = \left[\frac{x^4}{2} - \frac{17x^3}{3} + 20x^2\right]_0^4$$

$$= \left(128 - \frac{1088}{3} + 320\right) - (0)$$

$$= \underline{\underline{\frac{256}{3}}}$$