14. A curve C has equation $y = f(x)$ where

$$f(x) = -3x^2 + 12x + 8$$

(a) Write $f(x)$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

The curve C has a maximum turning point at M.

(b) Find the coordinates of M.

(2)

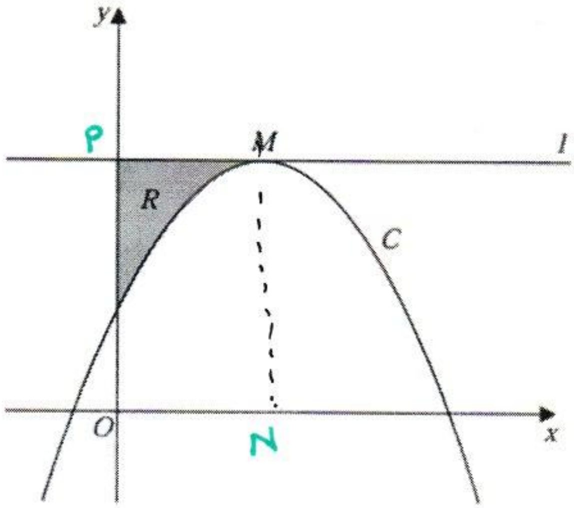

Figure 3

Figure 3 shows a sketch of the curve C.

The line l passes through M and is parallel to the x-axis.

The region R, shown shaded in Figure 3, is bounded by C, l and the y-axis.

(c) Using algebraic integration, find the area of R.

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(5)

(a) $f(x) = -3x^2 + 12x + 8 = -3(x^2 - 4x) + 8$

$= -3\{(x - 2)^2 - 4\} + 8 = -3(x-2)^2 + 20$

So $a = -3$, $b = -2$, $c = 20$

(b) The maximum value of $f(x)$ is at $x = 2$ when $-3(x-2)^2 = 0$

When $x = 2$, $y = c = 20$.

The coordinates of M are therefore $(2, 20)$.

otherwise this is -ve

(c) The area R is the area of rectangle ONMP (my marks on diag) minus the area under the curve from 0 to 2.

Area of rectangle $= 2 \times 20 = 40$.

Area under curve $= \int_0^2 (-3x^2 + 12x + 8) = \left[-x^3 + 6x^2 + 8x\right]_0^2$

$= (-8 + 24 + 16) - (0) = 32$

So area of R $= 40 - 32 = 8$