

11. The owners of a nature reserve decided to increase the area of the reserve covered by trees.

Tree planting started on 1st January 2005.

The area of the nature reserve covered by trees, $A \text{ km}^2$, is modelled by the equation

$$A = 80 - 45e^{-ct}$$

where c is a constant and t is the number of years after 1st January 2005.

Using the model,

- (a) find the area of the nature reserve that was covered by trees just before tree planting started.

(1)

On 1st January 2019 an area of 60 km^2 of the nature reserve was covered by trees.

- (b) Use this information to find a complete equation for the model, giving your value of c to 3 significant figures.

(4)

On 1st January 2020, the owners of the nature reserve announced a long-term plan to have 100 km^2 of the nature reserve covered by trees.

- (c) State a reason why the model is not appropriate for this plan.

2021

(1)

(a) Before planting $t = 0$ $e^0 = 1$ (anything⁰ = 1)
So $A = 80 - 45 = \underline{35 \text{ km}^2}$

(b) On 1st Jan 2019 $t = 14$ years. At this time $A = 60 \text{ km}^2$

So substituting

$$60 = 80 - 45e^{-c \times 14}$$

$$\Rightarrow 45e^{-14c} = 20$$

$$\Rightarrow e^{-14c} = \frac{20}{45} \quad \text{Take ln of both sides}$$

By definition $\ln(e^{-14c}) = -14c$

$$\text{So } -14c = \ln\left(\frac{20}{45}\right) = -0.81093$$

$$\text{Giving } c = 0.0579.$$

So our model can be written $A = 80 - 45e^{-0.0579t}$

(c) As $t \rightarrow \infty$ $e^{-0.0579t} \rightarrow 0$. At this time (?) the area approaches 80 km^2 . So the model cannot predict an area greater than 80 km^2 . The model is therefore inappropriate if 100 km^2 is required.