13.

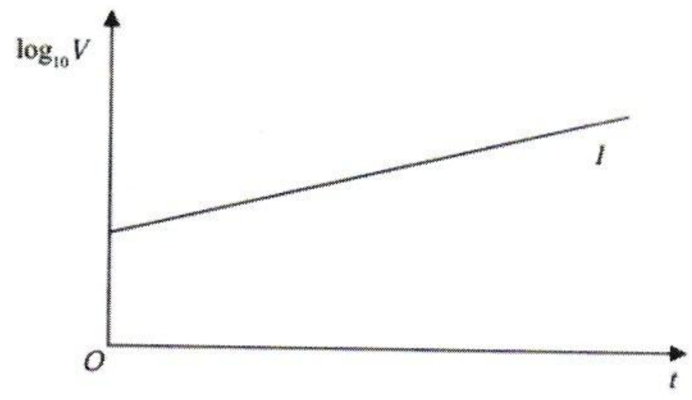

Figure 3

The value of a rare painting, £V, is modelled by the equation $V = pq^t$, where p and q are constants and t is the number of years since the value of the painting was first recorded on 1st January 1980.

The line l shown in Figure 3 illustrates the linear relationship between t and $\log_{10} V$ since 1st January 1980.

The equation of line l is $\log_{10} V = 0.05t + 4.8$

(a) Find, to 4 significant figures, the value of p and the value of q.

(4)

(b) With reference to the model interpret

(i) the value of the constant p,

(ii) the value of the constant q.

(2)

(c) Find the value of the painting, as predicted by the model, on 1st January 2010, giving your answer to the nearest hundred thousand pounds.

2018

(2)

$\log_{10} V = 0.05t + 4.8$

$V = 10^{(0.05t+4.8)} = 10^{4.8} \times 10^{0.05t}$ — $x^{a+b} = x^a \times x^b$

$= (10^{4.8}) \times (10^{0.05})^t$ — $x^{mn} = (x^m)^n$

So $p = 10^{4.8} = \underline{63100}$ and $q = 10^{0.05} = \underline{1.122}$ each 4 sig fig

(i) $V = p$ when $q^t = 1$ ie $t = 0$ So p is value at Jan 1 1980

(ii) q is the factor by which the value goes up each year. ie multiply by q each year.

(c) From 1980 to 2010 is 30 years so 30 x more!

$V = 63100 \times (1.122)^{30} = 63100 \times 31.61$

$= 1994,412$

$\simeq \underline{2,020,000}$