8. The temperature, θ°C, of a cup of tea t minutes after it was placed on a table in a room, is modelled by the equation

$$\theta = 18 + 65e^{-\frac{t}{8}} \qquad t \geqslant 0$$

Find, according to the model,

(a) the temperature of the cup of tea when it was placed on the table,

(1)

(b) the value of t, to one decimal place, when the temperature of the cup of tea was 35 °C.

(3)

(c) Explain why, according to this model, the temperature of the cup of tea could not fall to 15 °C.

(1)

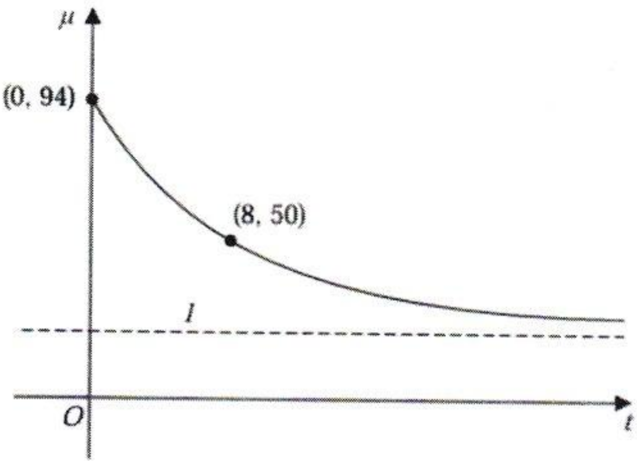

Figure 2

The temperature, μ°C, of a second cup of tea t minutes after it was placed on a table in a different room, is modelled by the equation

$$\mu = A + Be^{-\frac{t}{8}} \qquad t \geqslant 0$$

where A and B are constants.

Figure 2 shows a sketch of μ against t with two data points that lie on the curve.

The line l, also shown on Figure 2, is the asymptote to the curve.

Using the equation of this model and the information given in Figure 2

(d) find an equation for the asymptote l.

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(4)

(a) At $t = 0$, $e^{-t/8} = e^0 = 1$ So $\theta = 18 + 65 \times 1$

$= \underline{83°C}$

(b) When $\theta = 35$ $\quad 35 = 18 + 65e^{-t/8}$

so $e^{-t/8} = 17/65$

Take ln of both sides $-\frac{t}{8} = \ln\left(\frac{17}{65}\right) = -1.34$

giving $t = \underline{10.7 \text{ minutes}}$

(c) As $t \to \infty$ $e^{-t/8} \to 0$. It is never negative
So the minimum temperature of the tea is $\underline{18^\circ C}$

(d) $\mu = A + Be^{-t/8}$
when $t = 0$ $\mu = A + B \times 1 = A + B$
So from the graph $A + B = 94$ — (i)

When $t = 8$ $\mu = 50$
so $50 = A + Be^{-8/8}$
$= A + Be^{-1} = 50$ — (2)

Subtracting (2) from (1)
$B - Be^{-1} = 94 - 50 = 44$
$B(1 - e^{-1}) = 44$
$\Rightarrow$ $B = \dfrac{44}{(1-e^{-1})} = 69$ $\Rightarrow A = 25$

So $\mu = 25 + 69e^{-t/8}$
When $t = \infty$ $e^{-t/8} \to 0$ so $\mu \to 25$
Hence the equation for the asymptote is
$\underline{\mu = 25.}$ as a straight line // to the t axis.