10.

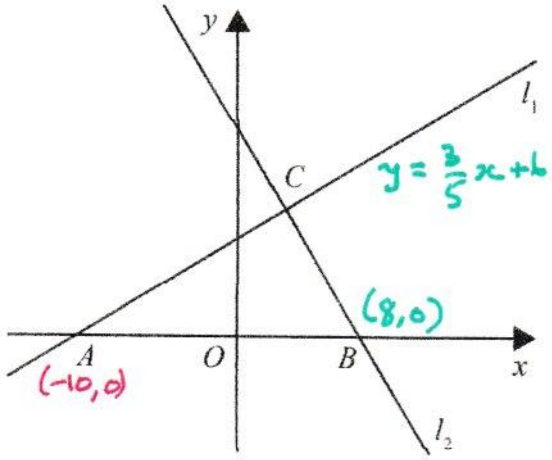

Figure 4

The line l_1 has equation $y = \frac{3}{5}x + 6$

The line l_2 is perpendicular to l_1 and passes through the point $B(8,0)$, as shown in the sketch in Figure 4.

(a) Show that an equation for line l_2 is

$$5x + 3y = 40$$

(3)

Given that

- lines l_1 and l_2 intersect at the point C
- line l_1 crosses the x-axis at the point A

(b) find the exact area of triangle ABC, giving your answer as a fully simplified fraction in the form $\frac{p}{q}$

(5)

(a) Gradient of $l_2 = -\frac{5}{3}$ as gradient of l_1 is $\frac{3}{5}$

So for l_2 $y = -\frac{5}{3}x + c$

But $y = 0$ when $x = 8$ as l_2 passes through B
$\Rightarrow c = 40/3$

So l_2 is $y = -\frac{5x}{3} + \frac{40}{3} \Rightarrow \underline{5x + 3y = 40}$ as required

(b) Point A is on l_1 when $y = 0$ so $x = -\frac{30}{3} = \underline{-10}$

Area of ABC $= \frac{1}{2}$ base $\times$ height $= \frac{1}{2}(AB) \times (y$ coord of C$)$

P 7 2 8 3 9 A 0 2 0 4 4

So we must find the y coordinate of c by finding the intersection of l_1 and l_2

$$l_1: \quad y = \frac{3}{5}x + 6$$

$$l_2: \quad y = -\frac{5}{3}x + \frac{40}{3}$$

So $\frac{3x}{5} + 6 = -\frac{5}{3}x + \frac{40}{3}$

$$\frac{(9+25)x}{15} = \frac{40-18}{3}$$

$$34x = 22 \times 5$$

$$\underline{x = \frac{55}{17}}$$

So y coordinate of $C = \frac{3}{5} \times \frac{55}{17} + 6$

$$= \frac{33}{17} + 6 \quad = \frac{135}{17}$$

So area of ABC $= \frac{1}{2}(8+10) \times \frac{135}{17}$

$$\underline{\underline{= \frac{1215}{17}}}$$