11.

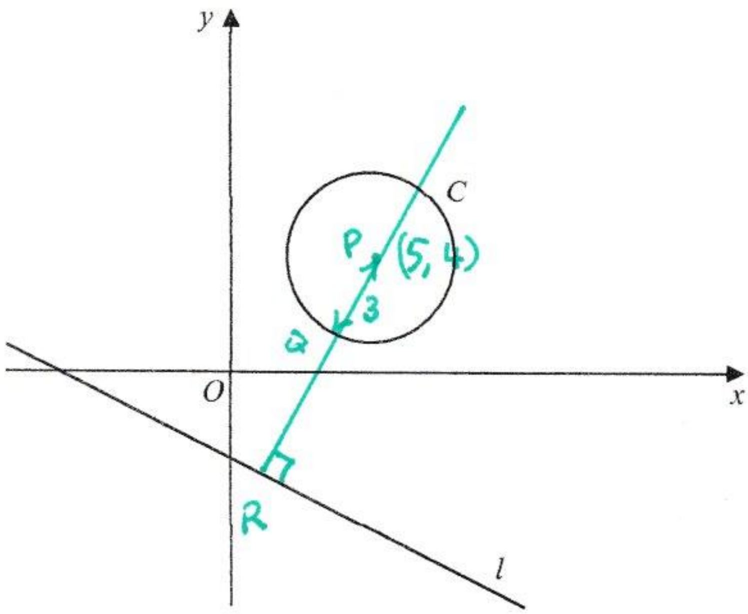

Figure 3

Figure 3 shows the circle C with equation

$$x^2 + y^2 - 10x - 8y + 32 = 0$$

and the line l with equation

$$2y + x + 6 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the radius of C.

(3)

(b) Find the shortest distance between C and l.

(5)

(a) C rewritten is

$$(x-5)^2 - 25 + (y-4)^2 - 16 + 32 = 0$$

or $(x-5)^2 + (y-4)^2 = 9$

(i) Coords of centre are $\underline{(5,4)}$

(ii) radius $= \sqrt{9} = \underline{3}$

Add data from (a) to diagram and draw in line of shortest distance ($\perp^r$ to l and $\perp^r$ to where it intersects C)
Call centre of circle P.

P 6 9 2 0 1 A 0 3 4 4 8

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Key to note. The green line QR is $\perp^r$ to C and so continues on to P the centre of the circle.
So the required length RQ = distance of R from centre MINUS the radius 3.

The key then is what are the coordinates of R.
Need to find the equation of the line RQP.

Let RQP have equation

$$y = mx + c$$

Gradient of $l = -\frac{1}{2}$ $\left(y = -\frac{1}{2}x - 6\right)$

So gradient of $RQP = 2$.

But RQP passes through $(5,4)$

So $y = 2x + c$

$4 = 10 + c$

$c = -6$

$\Rightarrow$ RPQ has equation $y = 2x - 6$.

Coordinates of R are the intersection of $y = 2x - 6$ and $y = -\frac{1}{2}x - 6$

Substituting $2x - 6 = -\frac{1}{2}x - 3$

$\frac{5x}{2} = 3 \Rightarrow x = \frac{6}{5}$ and so $y = 2x - 6$

$= \frac{12}{5} - 6$

$= \frac{-18}{5}$

So R is $\left(\frac{6}{5}, -\frac{18}{5}\right)$.

The distance RP is $\sqrt{\left(5 - \frac{6}{5}\right)^2 + \left(4 + \frac{18}{5}\right)^2}$

$= \frac{\sqrt{(25-6)^2 + (20+18)^2}}{5} = \frac{\sqrt{19^2 + 38^2}}{5}$

$= \frac{19}{5}\sqrt{1+4} = \frac{19\sqrt{5}}{5}$.

So the shortest distance $= \left(\frac{19\sqrt{5}}{5} - 3\right) \simeq 8.5 - 3 = \underline{5.5}$ to 1 decp^l