

6. The circle  $C$  has equation

$$x^2 + y^2 - 6x + 10y + k = 0$$

where  $k$  is a constant.

- (a) Find the coordinates of the centre of  $C$ .

(2)

Given that  $C$  does not cut or touch the  $x$ -axis,

- (b) find the range of possible values for  $k$ .

(3)

$$x^2 + y^2 - 6x + 10y + k = 0$$

$$(x-3)^2 - 9 + (y+5)^2 - 25 + k = 0$$

So centre is at  $(3, -5)$  and  $r^2 = 34 - k \Rightarrow r = \sqrt{34 - k}$

(b) The (radius)<sup>2</sup> must be  $> 0$  so  $34 - k > 0 \Rightarrow k < 34$

Also the circle must not touch or cut the  $x$ -axis  
so  $r <$  the distance of the centre from the  $x$ -axis  
(which is 5)

$$\text{So } \sqrt{34 - k} < 5$$

$$34 - k < 25$$

$$k > 9$$

So the range for  $k$  is  $9 < k < 34$ .

