9. A company started mining tin in Riverdale on 1st January 2019.

A model to find the total mass of tin that will be mined by the company in Riverdale is given by the equation

$$T = 1200 - 3(n - 20)^2$$

where T tonnes is the total mass of tin mined in the n years after the start of mining.

Using this model,

(a) calculate the mass of tin that will be mined up to 1st January 2020,

(1)

(b) deduce the maximum total mass of tin that could be mined,

(1)

(c) calculate the mass of tin that will be mined in 2023.

(2)

(d) State, giving reasons, the limitation on the values of n.

(2)

2019

(a) At 1st January 2020 $n = 1$ giving

$T = 1200 - 3(1-20)^2$

$= 117$ tonnes

(b) After 20 years $(n-20) = 0$ and then $T = 1200$ (when $n > 20$ the second term becomes negative)

(c) In 2023 we need to find the mass total mined before and the mass total after and subtract, ie.,

$T(n=5) - T(n=4)$

$= 1200 - 3(5-20)^2 - \{1200 - 3(4-20)^2\}$

$= 3(16^2) - 3(15^2) = 3(16^2 - 15^2) = 3 \times (16+15)(16-15)$

$= 3 \times 31 = 93$ tonnes

Calculus ok but could use difference of two squares

(d) T_{max} is reached after 20 years

The total mined cannot decrease, that does not make sense. So the model can only be valid up to $n = 20$ years