8. A lorry is driven between London and Newcastle.

In a simple model, the cost of the journey £C when the lorry is driven at a steady speed of v kilometres per hour is

$$C = \frac{1500}{v} + \frac{2v}{11} + 60$$

(a) Find, according to this model,

(i) the value of v that minimises the cost of the journey,

(ii) the minimum cost of the journey.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

(b) Prove by using $\frac{d^2C}{dv^2}$ that the cost is minimised at the speed found in (a)(i).

(2)

(c) State one limitation of this model.

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(1)

(a)(i) For a minimum cost $\frac{dc}{dv} = 0$

$$\frac{dc}{dv} = -1500v^{-2} + \frac{2}{11}$$

$\frac{dc}{dv} = 0$ when $\frac{2}{11} = \frac{1500}{v^2} \Rightarrow v^2 = 8250 \Rightarrow \underline{v = 90.8 \text{ km h}^{-1}}$

(ii) At this speed $C = \frac{1500}{90.8} + \frac{2 \times 90.8}{11} + 60$

$$\underline{= £93.03}$$

(b) $\frac{d^2c}{dv^2} = \frac{-2(-1500)}{v^3} = \frac{3000}{v^3}$ which is +ve for all +ve values of v (v being negative makes no sense)

Hence the turning point is a minimum as $\frac{d^2c}{dv^2}$ is +ve

(c) It is impractical to think the speed can be kept constant for an entire journey - and illegal as the journey takes more than 2 hours (London - Newcastle, assuming Newcastle on Tyne).