12. A company makes drinks containers out of metal.

The containers are modelled as closed cylinders with base radius r cm and height h cm and the capacity of each container is 355 cm^3

The metal used

- for the circular base and the curved side costs 0.04 pence/cm^2
- for the circular top costs 0.09 pence/cm^2

Both metals used are of negligible thickness.

(a) Show that the total cost, C pence, of the metal for one container is given by

$$C = 0.13\pi r^2 + \frac{28.4}{r}$$

(4)

(b) Use calculus to find the value of r for which C is a minimum, giving your answer to 3 significant figures.

(4)

(c) Using $\frac{d^2C}{dr^2}$ prove that the cost is minimised for the value of r found in part (b).

(2)

(d) Hence find the minimum value of C, giving your answer to the nearest integer.

(2)

(a) Cost of the top $= \pi r^2 \times 0.09$, Cost of bottom $= \pi r^2 \times 0.04$

So cost of top and bottom $= 0.13\pi r^2$

Volume $= \pi r^2 h = 355$

Area of metal in curved sides $= 2\pi r h$

$= 2\left(\frac{\pi r^2 h}{r}\right)$

$= 2 \times 355 / r.$

Cost of curved sides $= \frac{2 \times 355}{r} \times 0.04 = \frac{28.4}{r}$

Hence $C = 0.13\pi r^2 + \frac{28.4}{r}$

(b) $\frac{dC}{dr} = 0.26\pi r - \frac{28.4}{r^2} = 0$ for a minimum

$\Rightarrow r^3 = 28.4 / 0.26\pi \Rightarrow r = 3.26$

(c) $\frac{d^2C}{dr^2} = 0.26\pi + \frac{28.4 \times 2}{r^3}$ which is bound to be +ve

so answer to (c) is a minimum.

(d) When $r = 3.26$, $C = 0.13\pi(3.26^2) + \frac{28.4}{3.26} = 13p$

P 6 9 2 0 1 A 0 3 8 4 8