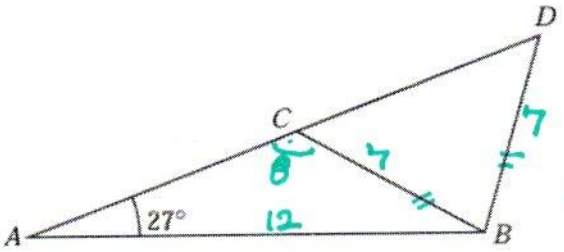
5.



Not to scale

θ = required angle

Figure 1

Figure 1 shows the design for a structure used to support a roof.

The structure consists of four steel beams, *AB*, *BD*, *BC* and *AD*.

Given $AB = 12$ m, $BC = BD = 7$m and angle $BAC = 27°$

(a) find, to one decimal place, the size of angle *ACB*.

(3)

The steel beams can only be bought in whole metre lengths.

(b) Find the minimum length of steel that needs to be bought to make the complete structure.

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(3)

(a) (Add dimensions to the diagram)

Use the sine rule in $\triangle ACB$

$$\frac{\sin\theta}{12} = \frac{\sin 27°}{7} \Rightarrow \sin\theta = \frac{12}{7}\sin 27° = 0.778$$

so $\theta = 51.1°$ if acute. But clearly θ is obtuse so

$$\theta = 180 - 51.1$$

$$= 128.9°$$

(b) Of the four beams we know $CB = 7$, $DB = 7$, $AB = 12$ so we need to find AD. To use the cosine rule in ABD we need $A\hat{B}D$

$A\hat{B}D = 180 - 27 - C\hat{D}B$. But $C\hat{D}B = B\hat{C}D$ (isoseles Δ)

$= 180 - \theta$

$= 51.1°$

So $A\hat{B}D = 101.9°$

No we can use the cosine rule:

$$AD^2 = 7^2 + 12^2 - 2\times 7\times 12\cos 101.9° = 227.64$$

$AD = 15.09$ m ← so 16 m needed

To get whole m lengths we need $7+7+12+16$

$= 42$m to buy.