4.

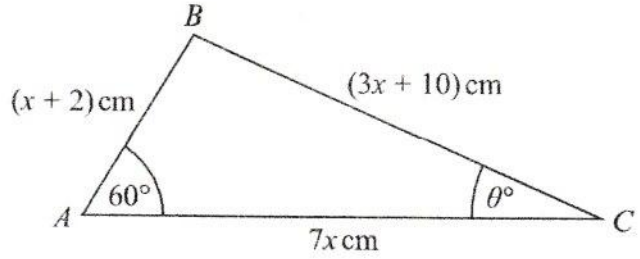

Figure 1

Figure 1 shows a sketch of triangle ABC with $AB = (x + 2)$ cm, $BC = (3x + 10)$ cm, $AC = 7x$ cm, angle $BAC = 60°$ and angle $ACB = \theta°$

(a) (i) Show that $17x^2 - 35x - 48 = 0$

(3)

(ii) Hence find the value of x.

(1)

(b) Hence find the value of θ giving your answer to one decimal place.

(2)

Here we have a triangle where all three sides are given in terms of x. There is also one angle. We need to use this data. The cosine rule involves all 3 sides and one angle

a (i) $BC^2 = AB^2 + AC^2 - 2AB \cdot AC \cos 60°$

$(3x+10)^2 = (x+2)^2 + (7x)^2 - 2(x+2)(7x) \times \frac{1}{2}$ $\quad \cos 60 = \frac{1}{2}$

$9x^2 + 60x + 100 = x^2 + 4x + 4 + 49x^2 - 7x^2 - 14x$

$x^2(9 - 1 - 49 + 7) + x(60 - 4 + 14) + 100 - 4 = 0$

$-34x^2 + 70x + 96 = 0$

Divide by 2 and change sign throughout $\underline{17x^2 - 35x - 48 = 0}$

(ii) This factorises $(x-3)(17x+16) = 0$. A -ve answer for x is meaningless so $\underline{x = 3}$

(b) Now we have all the sides and need another angle. So use sine rule

$\frac{\sin 60}{BC} = \frac{\sin \theta}{AB} \Rightarrow \frac{\sin 60}{19} = \frac{\sin \theta}{5}$

$\sin \theta = \frac{5}{19} \sin 60 = \frac{5}{19}\left(\frac{\sqrt{3}}{2}\right)$

$\underline{\theta = 13.2°}$ using calculator