13. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{1}{\cos\theta} + \tan\theta \equiv \frac{\cos\theta}{1 - \sin\theta} \qquad \theta \neq (2n+1)90° \quad n \in \mathbb{Z}$$

(3)

Given that $\cos 2x \neq 0$

(b) solve for $0 < x < 90°$

$$\frac{1}{\cos 2x} + \tan 2x = 3\cos 2x$$

giving your answers to one decimal place.

(5)

(a) $\frac{1}{\cos\theta} + \tan\theta = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \frac{1+\sin\theta}{\cos\theta}.$

We want a $(1-\sin\theta)$ in the denominator. So try top and bottom by $(1-\sin\theta)$ and see what happens?!

$= \frac{(1+\sin\theta)(1-\sin\theta)}{\cos\theta\,(1-\sin\theta)} = \frac{1-\sin^2\theta}{\cos\theta(1-\sin\theta)} = \frac{\cos^2\theta}{\cos\theta\,(1-\sin\theta)}$

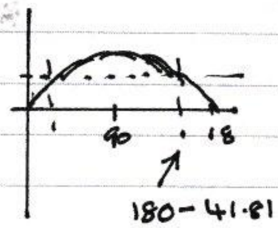
$= \underline{\underline{\frac{\cos\theta}{1-\sin\theta}}}$ as required

(b) Putting $\theta = 2x$ in LHS, equation to solve becomes

$\frac{\cos 2x}{1-\sin 2x} = 3\cos 2x$ as $x < 90$, $2x < 180$ and $x > 0$ $\cos 2x$ can never be zero so we can divide through

$\frac{1}{1-\sin 2x} = 3$

$1 - \sin 2x = 1/3 \Rightarrow \sin 2x = 2/3$



$\Rightarrow 2x = 41.81$ or 138.18

$\Rightarrow x = \underline{20.9}$ or $\underline{69.1}$