12. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$4\tan x = 5\cos x$$

can be written as

$$5\sin^2 x + 4\sin x - 5 = 0$$

(3)

(b) Hence solve, for $0 < x \leqslant 360°$

$$4\tan x = 5\cos x$$

giving your answers to one decimal place.

(4)

(c) Hence find the **number of solutions** of the equation

$$4\tan 3x = 5\cos 3x$$

in the interval $0 < x \leqslant 1800°$, explaining briefly the reason for your answer.

(2)

$4\tan x = 5\cos x$

$\Rightarrow \quad 4\dfrac{\sin x}{\cos x} = 5\cos x$

$\Rightarrow \quad 4\sin x = 5\cos^2 x = 5(1-\sin^2 x)$

Rearranging $\underline{5\sin^2 x + 4\sin x - 5 = 0}$

(b) The quadratic does not factorise

$\sin x = \dfrac{-4 \pm \sqrt{16+100}}{10} = \dfrac{-2 \pm \sqrt{29}}{5}$

The $-$ve sign gives a value for $\sin x$ which has a modulus > 1

So $\sin x = \dfrac{-2 + \sqrt{29}}{5}$ giving $x = \underline{42.6°}$

This is also an acceptable value in the 2nd quadrant $= 180 - 42.6° = \underline{137.4°}$

(c) There are 2 solutions for $\sin x$ in a $360°$ cycle – as above. So there are 6 solutions for $\sin 3x$. But in $1800°$ there are 5 cycles so the number of solutions is $5 \times 6 = \underline{30}$