

16. (i) Two non-zero vectors,  $\mathbf{a}$  and  $\mathbf{b}$ , are such that

$$|\mathbf{a} + \mathbf{b}| = |\mathbf{a}| + |\mathbf{b}|$$

Explain, geometrically, the significance of this statement.

(1)

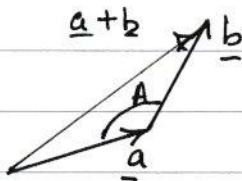
- (ii) Two different vectors,  $\mathbf{m}$  and  $\mathbf{n}$ , are such that  $|\mathbf{m}| = 3$  and  $|\mathbf{m} - \mathbf{n}| = 6$   
The angle between vector  $\mathbf{m}$  and vector  $\mathbf{n}$  is  $30^\circ$

Find the angle between vector  $\mathbf{m}$  and vector  $\mathbf{m} - \mathbf{n}$ , giving your answer, in degrees, to one decimal place.

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(4)

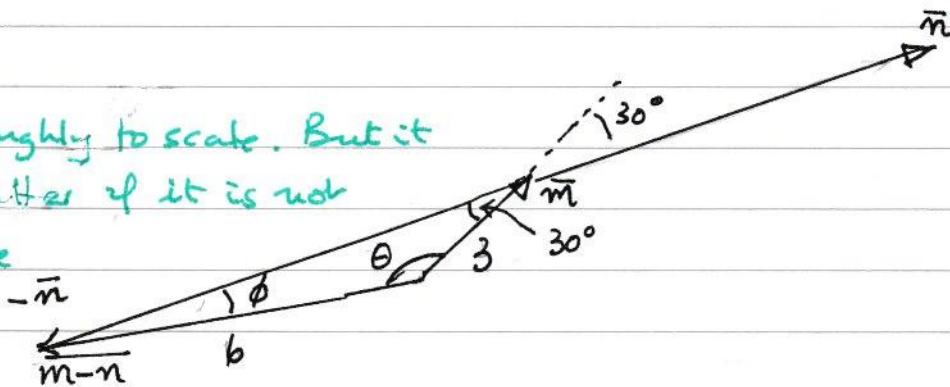
(i)



if the "length" of  $|\mathbf{a} + \mathbf{b}|$  is equal to the sum of the "lengths" of  $\mathbf{a}$  and  $\mathbf{b}$  i.e.  $|\mathbf{a}| + |\mathbf{b}|$ , the diagram shows that this is only possible if  $\mathbf{a}$  and  $\mathbf{b}$  are parallel.  
i.e. angle  $A = 180^\circ$

(ii)

The figure is roughly to scale. But it does not matter if it is not drawn so - the answer will come out ok.



It is clear  $|\mathbf{n}|$  must be quite large if  $|\mathbf{m} - \mathbf{n}| = 6$  and so the required angle  $\theta$  is also large (obtuse).

We cannot use sine rule directly to get  $\theta$  as we do not know  $|\mathbf{n}|$ . So use the sine rule to get  $\phi$  and then  $\theta = 180 - 30 - \phi$ .

$$\frac{3}{\sin \phi} = \frac{6}{\sin 30} \quad \text{so } \sin \phi = \frac{1}{2} \sin 30 = \frac{1}{4}$$

This gives  $\phi = 14.5^\circ$

$$\begin{aligned} \text{So required angle } \theta &= 180 - 30 - 14.5 \\ &= \underline{\underline{135.5^\circ}} \end{aligned}$$