4. [*In this question the unit vectors* **i** *and* **j** *are due east and due north respectively.*]

A stone slides horizontally across ice.

Initially the stone is at the point $A(-24\mathbf{i} - 10\mathbf{j})$ m relative to a fixed point O.

After 4 seconds the stone is at the point $B(12\mathbf{i} + 5\mathbf{j})$ m relative to the fixed point O.

The motion of the stone is modelled as that of a particle moving in a straight line at constant speed.

Using the model,

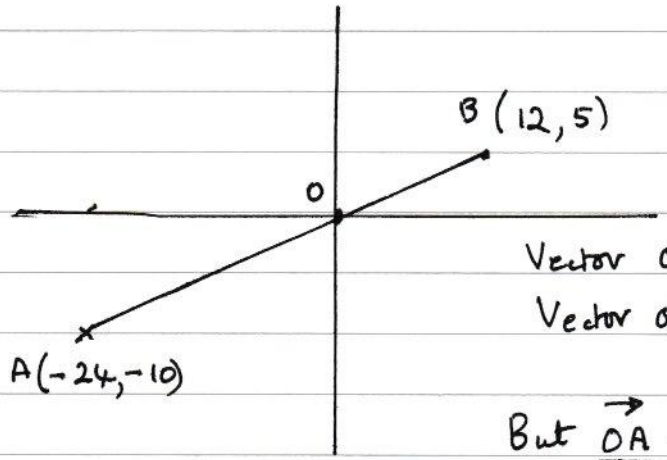
(a) prove that the stone passes through O,

(2)

(b) calculate the speed of the stone.

2021

(3)



Vector $OB = 12\underline{i} + 5\underline{j}$

Vector OA = − Vector $AO = (24\underline{i} + 10\underline{j})$

But $\overrightarrow{OA} = 2\overrightarrow{OB}$ so OA and OB are parallel. So AOB is a straight line. Hence stone must pass through O.

The distance $AB^2 = (24+12)^2 + (10+5)^2$

$= 36^2 + 15^2$

So $AB^2 = 1521$

$AB = 39$ m

So speed = Distance / time

$= \frac{39}{4} = 9.75$ m/s.