13. Relative to a fixed origin O

- point A has position vector $10\mathbf{i} - 3\mathbf{j}$
- point B has position vector $-8\mathbf{i} + 9\mathbf{j}$
- point C has position vector $-2\mathbf{i} + p\mathbf{j}$ where p is a constant

(a) Find $\overrightarrow{AB}$

(2)

(b) Find $\left|\overrightarrow{AB}\right|$ giving your answer as a fully simplified surd.

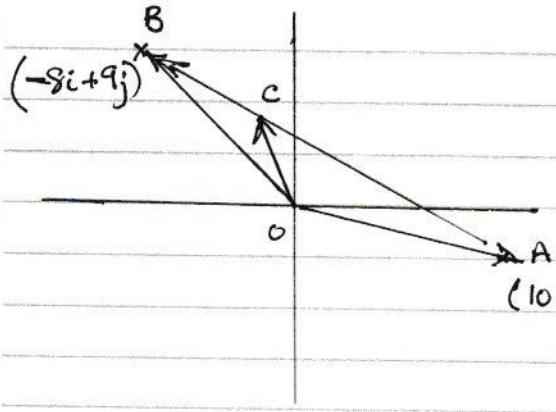
(2)

Given that points A, B and C lie on a straight line,

(c) (i) find the value of p,

(ii) state the ratio of the area of triangle AOC to the area of triangle AOB.

(3)



(a) $\overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB}$

$= -(10i - 3j) + (-8i + 9j)$

$= -18i + 12j$

(b) $|AB| = \sqrt{(-18)^2 + (12)^2}$

$= \sqrt{468}$ ← 36 is a factor

$= \sqrt{36 \times 13}$

$= 6\sqrt{13}$

(B) C lies on AB

Gradient of AB $= -\frac{12}{18} = -\frac{2}{3}$

This must also be the gradient of AC if C is on AB

Gradient of AC $= -\left(\frac{p+3}{12}\right) = \frac{\text{Change in } j}{\text{Change in } i}$

as C is at $(-2i + pj)$

So $-\left(\frac{p+3}{12}\right) = -\frac{2}{3}$

$p + 3 = \frac{24}{3} = 8$

$\Rightarrow \quad p = 5$