6. **In this question you should show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Using algebra, find all solutions of the equation

$$3x^3 - 17x^2 - 6x = 0$$

(3)

(b) Hence find all real solutions of

$$3(y-2)^6 - 17(y-2)^4 - 6(y-2)^2 = 0$$

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(3)

(a) $3x^3 - 17x^2 - 6x = 0$

$\Rightarrow \quad x(3x^2 - 17x - 6) = 0$

$\Rightarrow \quad x(3x+1)(x-6) = 0$

So the solutions are $x = 0$, $x = -\frac{1}{3}$ or $x = 6$

(b) If $x = (y-2)^2$ then

$3(y-2)^6 - 17(y-2)^4 - 6(y-2)^2 = 0$

becomes the equation just solved i.e.

$3x^3 - 17x^2 - 6x = 0$

When $x = 0$, $(y-2)^2 = 0 \Rightarrow y - 2 = 0 \Rightarrow y = 2$

When $x = -\frac{1}{3}$, $(y-2)^2 = -\frac{1}{3}$ so y is not real

When $x = 6$, $(y-2)^2 = 6$, $y - 2 = \sqrt{6}$ or $y - 2 = -\sqrt{6}$

so $y = 2 \pm \sqrt{6}$

Hence real solutions are $y = 2$, $y = 2 + \sqrt{6}$, $y = 2 - \sqrt{6}$