

15.

In this question you must show detailed reasoning.

Solutions relying on calculator technology are not acceptable.

The curve C_1 has equation $y = 8 - 10x + 6x^2 - x^3$

The curve C_2 has equation $y = x^2 - 12x + 14$

(a) Verify that when $x = 1$ the curves C_1 and C_2 intersect.

(2)

The curves also intersect when $x = k$.

Given that $k < 0$

(b) use algebra to find the exact value of k .

(5)

(a) When $x = 1$ y for $C_1 = 3$ $(8 - 10 + 6 - 1)$

y for $C_2 = 3$ $(1 - 12 + 14)$

So at $x = 1$ the curves intersect at $(1, 3)$.

(b) When the curves intersect

$$8 - 10x + 6x^2 - x^3 = x^2 - 12x + 14$$

Collecting terms and rearranging

$$x^3 - 5x^2 - 2x + 6 = 0$$

But we know $x = 1$ is a solution, so $(x - 1)$ must be a factor.

$$\begin{array}{r}
 x^2 + 4x - 6 \\
 x - 1 \overline{) x^3 - 5x^2 - 2x + 6} \\
 \underline{x^3 - x^2} \\
 -4x^2 - 2x \\
 \underline{-4x^2 + 4x} \\
 -6x + 6 \\
 \underline{-6x + 6} \\
 0
 \end{array}$$

So the second factor is $x^2 - 4x - 6$

This does not factorise so $x = \frac{4 \pm \sqrt{16 + 24}}{2}$

$$= 2 \pm \sqrt{4 + 6} = 2 \pm \sqrt{10}$$

But we are told to find the negative value of k

So $k = 2 - \sqrt{10}$

