11. $$f(x) = 2x^3 - 13x^2 + 8x + 48$$

(a) Prove that $(x - 4)$ is a factor of $f(x)$.

(2)

(b) Hence, using algebra, show that the equation $f(x) = 0$ has only two distinct roots.

(4)

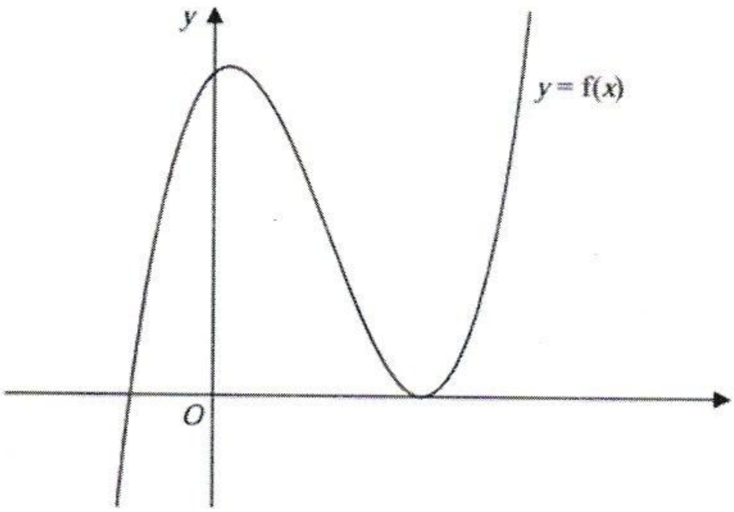

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$.

(c) Deduce, giving reasons for your answer, the number of real roots of the equation

$$2x^3 - 13x^2 + 8x + 46 = 0$$

(2)

Given that k is a constant and the curve with equation $y = f(x + k)$ passes through the origin,

(d) find the two possible values of k.

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(2)

(a) When $x = 4$ $f(x) = 128 - 13 \times 16 + 32 + 48$

$= 128 - 208 + 80 = \underline{0}$

Hence $(x-4)$ is a factor of $f(x)$.

(b) We need to find the remaining factor(s)

$$\begin{array}{r} 2x^2 - 5x - 12 \\ (x-4)\overline{)\,2x^3 - 13x^2 + 8x + 48} \\ 2x^2 - 8x^2 \\ \hline -5x^2 + 8x \\ -5x^2 + 20x \\ \hline -12x + 48 \\ -12x + 48 \\ \hline \end{array}$$

But $(2x^2 - 5x - 12)$ factorises to $(2x+3)(x-4)$

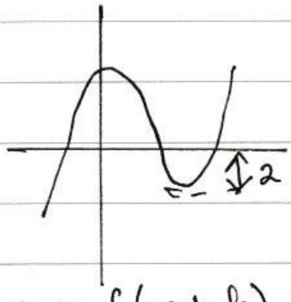
So $f(x) = (x-4)(2x+3)(x-4)$ So there are only two roots to $f(x) = 0$ ie $\underline{\underline{4 \text{ and } -3/2}}$

(c) if $g(x) = 2x^3 - 13x^2 + 8x + 46$

$g(x) = f(x) - 2$

Thus $g(x)$ is shifted by 2 units down along y axis.

So from shape given there will now be $\underline{3}$ solutions



(d) $y = f(x+k)$.

The original graph needs to be shifted in the $\underline{+ve}$ x direction by $3/2$ units for the solution at $-3/2$ to now be at the origin. In this case k must be $\underline{\underline{-3/2}}$

For the "touching" solution at $x = 4$ the graph needs to shifted in the $\underline{-ve}$ x direction by 4 units.

In this case $\underline{\underline{k = 4}}$

So the values of k are $\underline{-3/2 \text{ and } 4}$