7. (a) Factorise completely $9x - x^3$

(2)

The curve C has equation

$$y = 9x - x^3$$

(b) Sketch C showing the coordinates of the points at which the curve cuts the x-axis.

(2)

The line l has equation $y = k$ where k is a constant.

Given that C and l intersect at 3 distinct points,

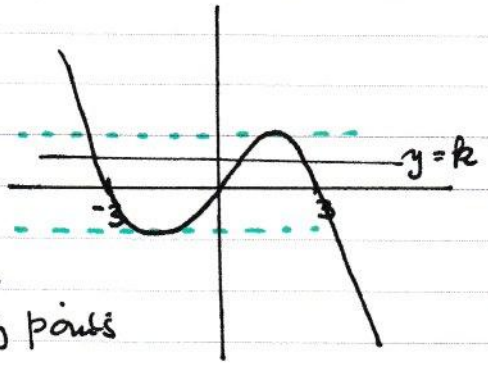
(c) find the range of values for k, writing your answer in set notation.

Solutions relying on calculator technology are not acceptable.

(3)

(a) $9x - x^3 = x(9 - x^2) = x(3 - x)(3 + x)$

(b) The equation $9x - x^3 = 0$ has solutions $0, -3, 3$.
Also for $y = 9x - x^3$ the x^3 term is $-ve$ so
for $-ve\ x$, y gets large and $+ve$
for $+ve\ x$, y gets large and $-ve$
Hence the graph



(c) Between the dotted lines $y = k$ intersects the graph at 3 points.
So we need to find the y value at the turning points

$$\frac{dy}{dx} = 9 - 3x^2$$

$\frac{dy}{dx} = 0$ when $x = \pm\sqrt{3}$.

At these values of x $\quad y = 9\sqrt{3} - 3\sqrt{3} = 6\sqrt{3}$. ($(\sqrt{3})^3$)

So k must lie between these values

$$\{k \in R : -6\sqrt{3} < k < 6\sqrt{3}\}$$