2. $$f(x) = 2x^3 + 5x^2 + 2x + 15$$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$.

(2)

(b) Find the constants a, b and c such that

$$f(x) = (x + 3)(ax^2 + bx + c)$$

(2)

(c) Hence show that $f(x) = 0$ has only one real root.

(2)

(d) Write down the real root of the equation $f(x - 5) = 0$

(1)

(a) $f(-3) = -54 + 45 - 6 + 15 = 0$
So $(x+3)$ is a factor

(b) Do long division to get the second factor

$$\begin{array}{r|l} & 2x^2 - x + 5 \\ \hline (x+3) & 2x^3 + 5x^2 + 2x + 15 \\ & 2x^3 + 6x^2 \\ \hline & \quad -x^2 + 2x \\ & \quad -x^2 - 3x \\ \hline & \qquad 5x + 15 \\ & \qquad 5x + 15 \end{array}$$

So $f(x) = (x+3)(2x^2 - x + 5)$ $\quad a = 2, b = -1, c = 5$

(c) For there to be one root $2x^2 - x + 5$ should have no real roots. The determinant is $b^2 - 4ac = 1 - 40 = -39$
So there are no real roots to $(2x^2 - x + 5)$, so $f(x)$ has only one real root. The real root is $x = -3$.

(d) $f(x-5)$ gives a translation of 5 in the +ve x direction
So the solution of $f(x-5)$ is $-3+5 = \underline{\underline{2}}$

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