

8.

$$g(x) = (2 + ax)^8 \quad \text{where } a \text{ is a constant}$$

Given that one of the terms in the binomial expansion of $g(x)$ is $3402x^5$

(a) find the value of a .

(4)

Using this value of a ,

(b) find the constant term in the expansion of

$$\left(1 + \frac{1}{x^4}\right)(2 + ax)^8$$

2021
(3)

$$\begin{aligned} \text{(a) The } x^5 \text{ term is } 2^{\binom{8-5}{1}} (ax)^5 \times \binom{8}{5} &\leftarrow \left(\frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} = 56\right) \\ &= 2^3 a^5 x^5 \times 56 \\ &= 448 a^5 x^5 \end{aligned}$$

This gives a from

$$448 a^5 = 3402$$

$$a^5 = \frac{3402}{448} = \frac{1701}{224}$$

7 is a factor top & bottom

$$= \frac{243}{32}$$

$$a = \sqrt[5]{\frac{243}{32}} = \frac{3}{2} \quad (3^5 = 243 \text{ and } 2^5 = 32)$$

There are two constant terms $\left(1 + \frac{1}{x^4}\right) \left(2^8 \dots \dots 2^4 a^4 x^4 \times 70 \dots \dots\right)$
↑ ↑
cancels

the two terms are 2^8 and $2^4 a^4 \cdot 70$
 So the total constant term is

$$2^8 + 2^4 \times \left(\frac{3}{2}\right)^4 \times 70$$

$$= 2^8 + 3^4 \times 70 = 256 + 5670$$

$$= \underline{5926}$$