10.

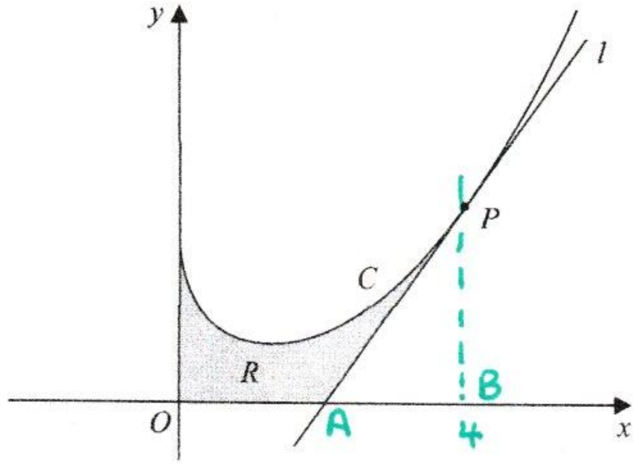

Figure 2

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 2 shows a sketch of part of the curve C with equation

$$y = \frac{1}{3}x^2 - 2\sqrt{x} + 3 \qquad x \geqslant 0$$

The point P lies on C and has x coordinate 4

The line l is the tangent to C at P.

(a) Show that l has equation

$$13x - 6y - 26 = 0$$

(5)

The region R, shown shaded in Figure 2, is bounded by the y-axis, the curve C, the line l and the x-axis.

(b) Find the exact area of R.

(5)

(a) To find l we need its gradient = gradient of C at $x=4$

$\frac{dy}{dx} = \frac{2x}{3} - \frac{2}{2\sqrt{x}}$. When $x=4$, $\frac{dy}{dx} = \frac{8}{3} - \frac{1}{2} = \frac{13}{6}$

So l has form $y = \frac{13}{6}x + c$ $\longrightarrow (1)$

But this passes through C. y coord at C is $\frac{16}{3} - 2\cdot 2 + 3 = \frac{13}{3}$

So putting $P = (4, {}^{13}/_{3})$ in $y = \frac{13}{6}x + c$ gives $c = -\frac{13}{3}$

Putting this in (1) and multiplying by 6 gives $6y = 13x - 26$

or $13x - 6y - 26 = 0$

(b) Add construction lines to diagram to make work and explanation easier

Area required = Area under graph between 0 and MINUS area of Δ PAB.

To find Δ A is at point on l where $y=0$

so $13x-6y-26=0 \Rightarrow 13x=26 \Rightarrow x=2$.

So area PAB $= \frac{1}{2} \times 2 \times (y \text{ coord of } P)$

$= \frac{1}{2} \times 2 \times \frac{13}{3} = \underline{\underline{\frac{13}{3}}}$

Area under graph $= \int_0^4 \left(\frac{1}{3}x^2 - 2\sqrt{x} + 3\right) dx$

$= \left[\frac{x^3}{9} - \frac{2x^{3/2}}{(3/2)} + 3x\right]_0^4$

$= \frac{1}{9}(4^3) - \frac{4}{3}(4)^{3/2} + 12$

$= \frac{64}{9} - \frac{32}{3} + 12 = \underline{\underline{\frac{76}{9}}}$

So area R $= \frac{76}{9} - \frac{13}{3} = \frac{76}{9} - \frac{39}{9} = \underline{\underline{\frac{37}{9}}}$