5. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

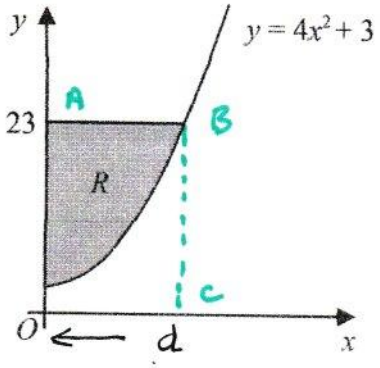

Figure 2

The finite region R, shown shaded in Figure 2, is bounded by the curve with equation $y = 4x^2 + 3$, the y-axis and the line with equation $y = 23$

Show that the exact area of R is $k\sqrt{5}$ where k is a rational constant to be found.

(5)

Required area = area of rectangle OABC − area under graph from 0 − d.

To find d

$23 = 4x^2 + 3 \Rightarrow d = \sqrt{5}$ (must take +ve root)

Area of rectangle $= 23d = 23\sqrt{5}$

Area under graph $= \int_0^{\sqrt{5}} (4x^2 + 3)\,dx$

$= \left[\frac{4x^3}{3} + 3x\right]_0^{\sqrt{5}}$

$= \frac{4}{3}(\sqrt{5})^3 + 3\sqrt{5} = \sqrt{5}\left(\frac{20}{3} + 3\right)$

$= \frac{29\sqrt{5}}{3}$

So required area $= \sqrt{5}\left\{23 - \frac{29}{3}\right\} = \frac{40}{3}\sqrt{5}$

So $R = k\sqrt{5}$ where $k = \frac{40}{3}$