16. A curve has equation $y = f(x)$, $x \geqslant 0$

Given that

- $f'(x) = 4x + a\sqrt{x} + b$, where a and b are constants
- the curve has a stationary point at $(4,3)$
- the curve meets the y-axis at -5

find $f(x)$, giving your answer in simplest form.

(6)

$f'(x) = 4x + a\sqrt{x} + b$

$f'(x) = 0$ at $(4,3)$ as stationary point

$\Rightarrow 16 + 2a + b = 0$ — (1)

Also $f(x)$ passes through $(0,-5)$

$f(x) = \int f'(x)\,dx = 2x^2 + \frac{2}{3}ax^{3/2} + bx + c$

Using ($f(x) = -5$ when $x = 0$) $\Rightarrow c = -5.$

So $f(x) = 2x^2 + \frac{2}{3}ax^{3/2} + bx - 5$

Since $(4,3)$ is a stationary point $f(x)$ must pass through $(4,3)$

giving $3 = 32 + \frac{2}{3}a(4)^{3/2} + 4b - 5$

$= 32 + \frac{16}{3}a + 4b - 5$

or $\frac{16}{3}a + 4b = -24$ — (2)

From (1) $8a + 4b = -64$ — (3)

Subtracting (2) from (3)

$\frac{24a}{3} - \frac{16a}{3} = -40 \Rightarrow \frac{8a}{3} = -40 \Rightarrow a = -15$

Substituting in (1) $b = -16 + 30 = 14.$

$\underline{\text{So } f(x) = 2x^3 - 10x^{3/2} + 14x - 5.}$