5. The mass, A kg, of algae in a small pond, is modelled by the equation

$$A = pq^t$$

where p and q are constants and t is the number of weeks after the mass of algae was first recorded.

Data recorded indicates that there is a linear relationship between t and $\log_{10} A$ given by the equation

$$\log_{10} A = 0.03t + 0.5$$

(a) Use this relationship to find a complete equation for the model in the form

$$A = pq^t$$

giving the value of p and the value of q each to 4 significant figures.

(4)

(b) With reference to the model, interpret

(i) the value of the constant p,

(ii) the value of the constant q.

(2)

(c) Find, according to the model,

(i) the mass of algae in the pond when $t = 8$, giving your answer to the nearest 0.5 kg,

(ii) the number of weeks it takes for the mass of algae in the pond to reach 4 kg.

(3)

(d) State one reason why this may not be a realistic model in the long term.

(1)

(a) $A = pq^t \Rightarrow \log_{10} A = \log_{10} p + t\log_{10} q$

Comparing with given data $\log_{10} p = 0.5$, $\log_{10} q = 0.03$

So $\underline{p = 10^{0.5} = 3.162}$ and $\underline{q = 10^{0.03} = 1.072}$

(b) $A = 3.162 \times (1.072^t)$ when $t = 0$, $1.072^0 = 1$ so $A = 3.162$

So p is the value of A at the start of the moddling.

q is the increment by which A is multiplied each year.

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(c)(i) When $t = 0$

$$A = 3.162 \times 1.072^{8}$$

$= 5.5$ kg to nearest 0.5 kg

(ii) When $A = 4$

$$4 = 3.162 \times 1.072^{t}$$

Take $\log_{10}$ of both sides

$$\log_{10} 4 = 0.5 + t \times 0.03$$

Remember Part A

$p = 10^{0.5}$ $q = 10^{0.03}$

$$t = \frac{0.602 - 0.5}{0.03}$$

$= 3.4$ weeks

(d) The pond will eventually become saturated with algae.