9. Using the laws of logarithms, solve the equation

$$2\log_5(3x-2) - \log_5 x = 2$$

(5)

$$2\log_5(3x-2) - \log_5 x = 2 \quad \text{— line 1}$$

$$\log_5(3x-2)^2 - \log_5 x = 2 \quad \text{— line 2}$$

$$\log\left\{\frac{(3x-2)^2}{x}\right\} = 2$$

$$\Rightarrow \frac{(3x-2)^2}{x} = 5^2 = 25$$

$$\Rightarrow (3x-2)^2 = 25x \quad\Rightarrow\quad 9x^2 - 12x + 4 = 25x$$

$$\Rightarrow 9x^2 - 37x + 4 = 0$$

$$\Rightarrow (9x-1)(x-4) = 0$$

So $x = \frac{1}{9}$ or 4. BUT when $x = \frac{1}{9}$, $3x - 2 < 0$

and you cannot have the log of a -ve number.

So the only acceptable answer is $x = 4$.

(Not part of answer). How come we get $\frac{1}{9}$ then?
If you put $\frac{1}{9}$ into line 2 above

$$\log_5\left(\frac{3}{9}-2\right)^2 - \log\left(\frac{1}{9}\right)$$

$$= \log_5\left(-\frac{5}{3}\right)^2 - \log\left(\frac{1}{9}\right)$$

$$= \log_5\left(\frac{25}{9}\right) - \log_5\frac{1}{9}$$

$$= \log_5\left(\frac{25}{9} \times \frac{9}{1}\right) = \log_5(25) = 2! \quad \text{it works!}$$

But line 1 does NOT work.
You should now be able to see what has happened if you look carefully at line 1 compared to line 2.