

11. The height, h metres, of a plant, t years after it was first measured, is modelled by the equation

$$h = 2.3 - 1.7e^{-0.2t} \quad t \in \mathbb{R} \quad t \geq 0$$

Using the model,

- (a) find the height of the plant when it was first measured, (2)
- (b) show that, exactly 4 years after it was first measured, the plant was growing at approximately 15.3 cm per year. (3)

According to the model, there is a limit to the height to which this plant can grow.

- (c) Deduce the value of this limit. (1)

(a) $h = 2.3 - 1.7e^{-0.2t}$
when first measured $t = 0$, $e^{-0.2 \times 0} = 1$

$$h = 2.3 - 1.7 = \underline{0.6 \text{ m}}$$

(b) $\frac{dh}{dt} = (-1.7)(-0.2)e^{-0.2t} = 0.34e^{-0.2t}$

At $t = 4$, $\frac{dh}{dt} = 0.34e^{-0.8} = 0.153 \text{ m yr}^{-1}$
 $= \underline{15.3 \text{ cm yr}^{-1}}$

(c) when $t \rightarrow \infty$, $e^{-0.2t} \rightarrow 0$ so the ultimate height is $\underline{2.3 \text{ m}}$.

