

9. (a) Given that $p = \log_3 x$, where $x > 0$, find in simplest form in terms of p ,

(i) $\log_3\left(\frac{x}{9}\right)$

(ii) $\log_3(\sqrt{x})$

(2)

(b) Hence, or otherwise, solve

$$2\log_3\left(\frac{x}{9}\right) + 3\log_3(\sqrt{x}) = -11$$

giving your answer as a simplified fraction.

Solutions relying on calculator technology are not acceptable.

(4)

(a) (i) $\log_3\left(\frac{x}{9}\right) = \log_3 x - \log_3 9 = \underline{p-2}$

(ii) $\log_3(\sqrt{x}) = \log_3 x^{1/2} = \frac{1}{2}\log_3 x = \underline{p/2}$

(b) $2\log_3\left(\frac{x}{9}\right) + 3\log_3(\sqrt{x}) = -11$

Write in terms of p

$$2(p-2) + 3\frac{p}{2} = -11$$

$$2p - 4 + \frac{3p}{2} = -11$$

$$\frac{7p}{2} = -7 \Rightarrow \frac{p}{2} = -1$$

$$p = -2$$

But p is given as $\log_3 x$

So $\log_3 x = -2$

$$x = 3^{-2}$$

$$= \frac{1}{9}$$

