8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The air pressure, P kg/cm^2, inside a car tyre, t minutes from the instant when the tyre developed a puncture is given by the equation

$$P = k + 1.4e^{-0.5t} \quad t \in \mathbb{R} \quad t \geqslant 0$$

where k is a constant.

Given that the initial air pressure inside the tyre was 2.2 kg/cm^2

(a) state the value of k.

(1)

From the instant when the tyre developed the puncture,

(b) find the time taken for the air pressure to fall to 1 kg/cm^2
Give your answer in minutes to one decimal place.

(3)

(c) Find the rate at which the air pressure in the tyre is decreasing exactly 2 minutes from the instant when the tyre developed the puncture.
Give your answer in kg/cm^2 per minute to 3 significant figures.

(2)

(a) $P = k + 1.4e^{-0.5t}$ When $t = 0$ $P = k + 1.4$

giving $k = 2.2 - 1.4$

$= 0.8$ kgcm^{-2}

(b) $P = 1$ here so $1 = 0.8 + 1.4e^{-0.5t}$

$\frac{0.2}{1.4} = e^{-0.5t}$

Take ln of both sides $\ln\left(\frac{1}{7}\right) = -0.5t \Rightarrow t = 3.9$ min

(c) $P = 0.8 + 1.4e^{-0.5t}$

$\frac{dP}{dt} = 1.4(-0.5)e^{-0.5t}$

$\frac{d}{dx}(e^{ax}) = ae^{ax}$

When $t = 2$

$\frac{dP}{dt} = -0.7e^{-1} = -\frac{0.7}{e} = -0.258$ kgcm^{-2}/min

$-$ve sign shows decreasing at rate 0.258 kgcm^{-2}/min.