

15. Given $n \in \mathbb{N}$, prove that $n^3 + 2$ is not divisible by 8

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(4)

In problems like this it is often necessary to treat even and odd numbers separately.

n even: so $n = 2m$ $m \in \mathbb{N}$

$$n^3 + 2 = 8m^3 + 2$$

$$(n^3 + 2)/8 = m^3 + 1/4$$

m^3 is a whole number

so clearly $m^3 + 1/4$ is not a whole number.

n odd so $n = 2m + 1$

$$(2m + 1)^3 + 2 = 8m^3 + 12m^2 + 6m + 3$$

$$= 2(4m^3 + 6m^2 + 3m) + 3$$

This is an odd number

so is not divisible by 8.

even makes odd