13. (a) Prove that for all positive values of a and b

$$\frac{4a}{b} + \frac{b}{a} \geqslant 4$$

(4)

(b) Prove, by counter example, that this is not true for all values of a and b.

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(1)

(a) Suppose truth (if not true then an error will show)

IF $\quad \frac{4a}{b} + \frac{b}{a} \geqslant 4$

$$\frac{4a^2 + b^2}{ab} \geqslant 4$$

a & b are +ve so ab is +ve so we can multiply by ab without worrying about the direction of the greater sign

$$4a^2 + b^2 \geqslant 4ab$$

or $\quad 4a^2 - 4ab + b^2 \geqslant 0$

$$(2a - b)^2 \geqslant 0$$

This is always true as $(2a-b)^2$ is positive or zero

Hence the original proposition is true.

(b) It will not be true if a or b is −ve

eg $\quad a = 1 \quad b = -1$

$\frac{4}{-1} + \frac{-1}{1} = -5$ which is not greater than 4