17. In this question p and q are positive integers with $q > p$

Statement 1: $q^3 - p^3$ is never a multiple of 5

(a) Show, by means of a counter example, that Statement 1 is **not** true.

(1)

Statement 2: When p and q are consecutive **even** integers $q^3 - p^3$ is a multiple of 8

(b) Prove, using algebra, that Statement 2 is true.

(4)

(a) Let $q = 6$ and $p = 1$

$q^3 - p^3 = 6^3 - 1^3$

$= 216 - 1 = 215$ which is a multiple of 5

So the statement is not true

(b) Let $p = 2n$ and then $q = 2n+2$

$q^3 - p^3 = (2n+2)^3 - (2n)^3$

Using Binomial (or do the expansion)

$= 8n^3 + 24n^2 + 24n + 8 - 8n^3$

$= 24n^2 + 24n + 8$

$= 8(3n^2 + 3n + 1)$

This is a multiple of 8.

(Aside any (even number)^3 is a multiple of 8 as it is $(2n)^3 = 8n^3$. The difference then between any different even numbers is also a multiple of 8. But we are asked to use algebra hence the above method).

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA