3. Movement in a Circle

In this essay you will see that:

- objects *can* accelerate without any change of speed
- there is *no such thing* as centrifugal force
- you are *not* thrown outwards when a car goes a bit fast round a corner

If you are surprised by any of these things, which is likely, then there will be some rethinking to do. It is the purpose of this essay to convince you that these statements are correct. The starting point may seem a bit irrelevant, but please bear with it. This essay is one of the hardest you will meet. Don't be put off.

Vectors

In the last section we said that, in science, velocity and acceleration have direction, as well as size, in order to specify them. They are examples of *vectors*. Distance is not a vector but there is a quantity called **displacement** which is. It is distance in a particular direction. Here is an example.

You are on a walk in the hills intending to get from the pub at A to the pub at B. The mist comes down and the compass you are using is a bit faulty and you actually end up at greasy café at C. Fig 3.1 shows a map, without the roads and contour lines.

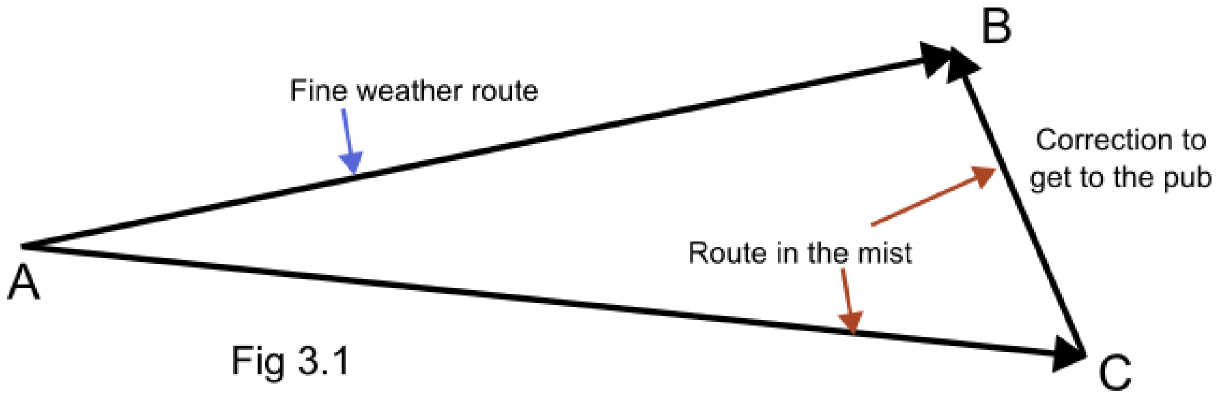

Fig 3.1

So you have actually travelled a distance of 5 miles in a direction from A to C. The line with an arrow joining A to C represents a distance of 5 miles in the AC direction. So this is a *vector* quantity, it has size and direction, and is called the **displacement.** But you really wanted to travel along the displacement vector A to B.

To correct the error, now that the mist has lifted, you need to travel along a displacement vector C to B as shown in the diagram. So had the weather been fine you would have gone A->B directly. But you achieved the same thing by A->C and C->A. So the vectors A->C and C->A **add** up to A->B. You start at A and end up at B but by different routes. We can write this mathematically as:

A->B = A->C + C->B

This is fairly obvious for the displacement vector, but it is also true of vectors like velocity, acceleration and force. They all obey this same addition rule. Start by representing them as lines with a length proportional to their size and make the lines point in their direction. Here is an example with velocity.

An oarsman is rowing across a river. He is aiming for the opposite bank and he rows at 3 m/s. The river itself is flowing along at 4 m/s. If you are looking from the bank, how fast is he actually travelling and in what direction is the boat moving? We can answer this using the idea of adding vectors, but this time they are velocity vectors. Fig 3.2 shows the velocity vectors.

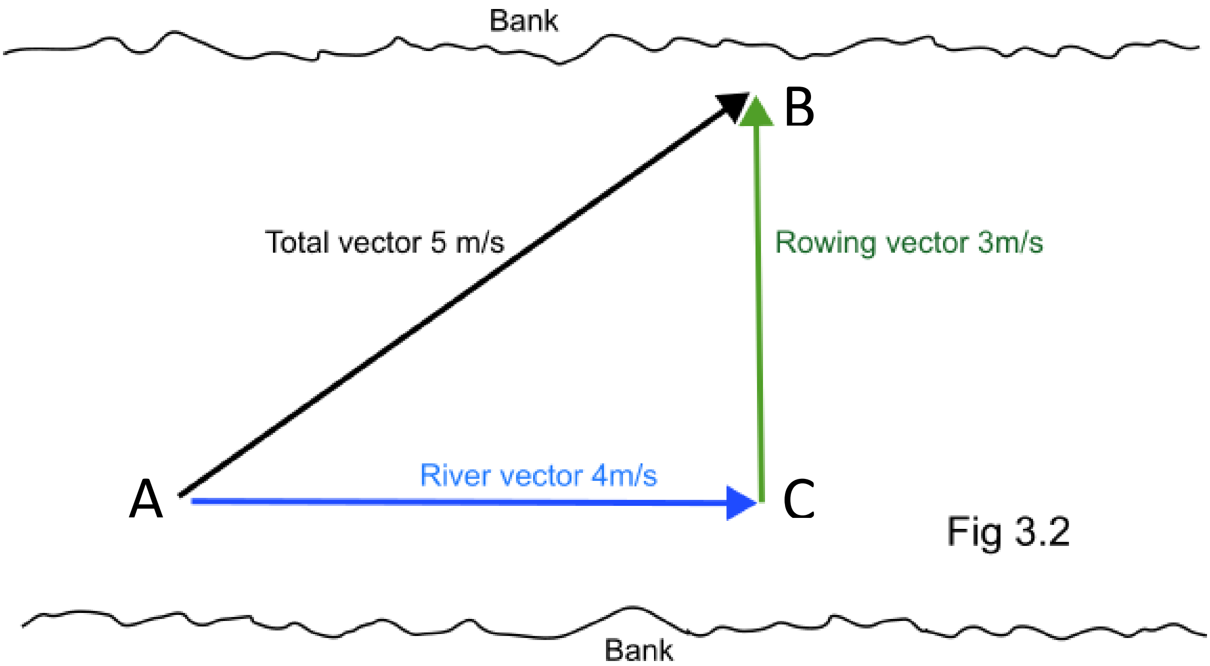

Fig 3.2

Just as in the case of displacement we add the vectors by completing the triangle ABC.

You can see from the picture that his speed in not 3 + 4 = 7 m/s as he is not rowing in the same direction as the river. If you draw the picture to scale and measure the length of AB (or if you know Pythagoras theorem) you will see that the speed is actually 5 m/s. If you know the width of the river and you did a bit of maths you could work out where he would hit the opposite bank. But we don't need to do that. The point we are making is that with vectors you add them up using the same triangle method as just described; i.e. In the diagram below:

Vector A->B = Vector A->C + Vector C->B whatever the vectors represent, as in Fig 3.3

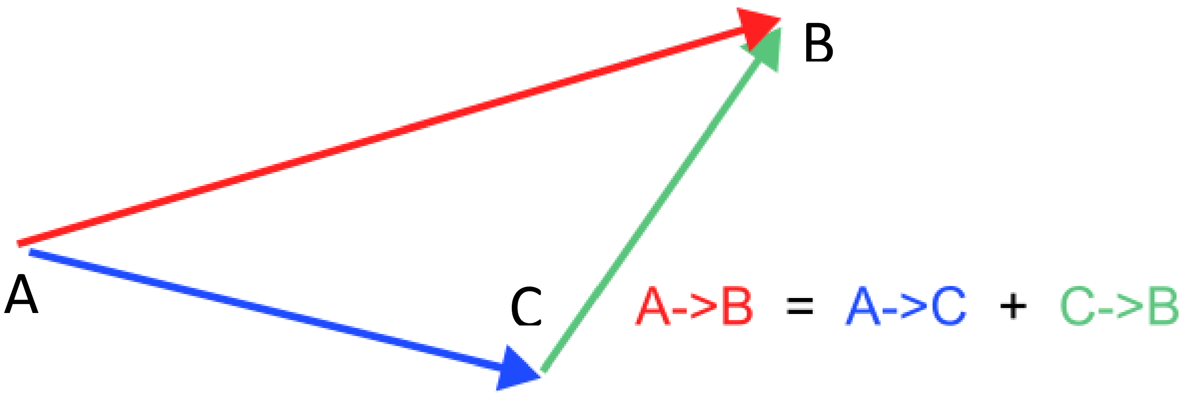

Fig 3.3

Movement in a circle

Driving round a bend in a car, a satellite orbiting the Earth, the Earth orbiting the sun, are all examples of something moving in a circle – maybe just part of a circle and perhaps not an exact circle – but near enough. So we can generalise and think of something moving in a circle without worrying what it is. There will be some concrete examples later.

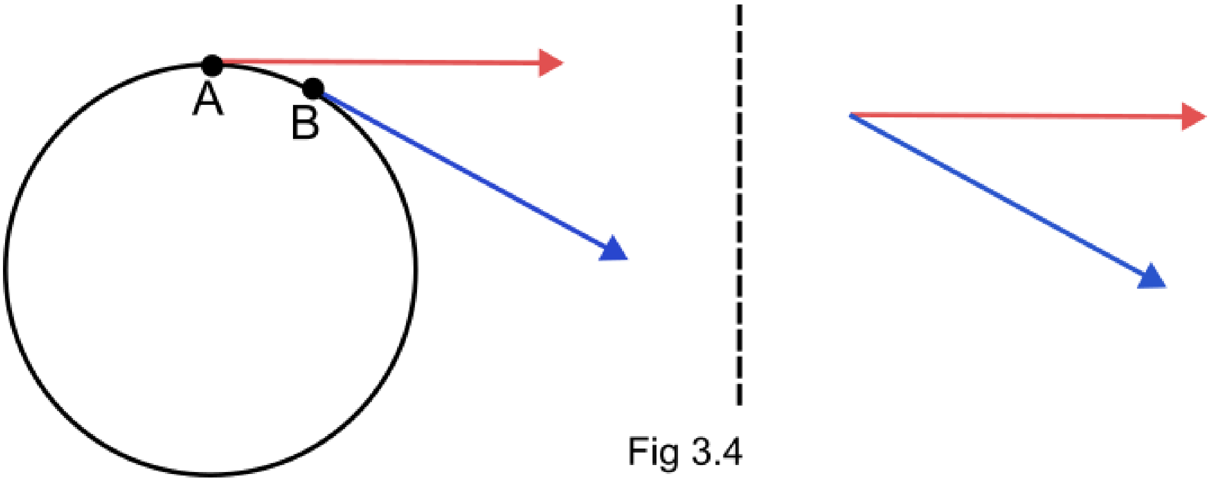

Fig 3.4

In Fig 3.4, on the left of the dotted line, the arrow at A represents the velocity at A and is shown in red. A short time later when the object has moved to B the velocity has changed direction and the velocity vector is shown in blue. The diagram on the right puts these two arrows next to one another. Now think back to the walk in the mist and ending up at the greasy café. To get back where you wanted to be you had to ADD another walk from C to B. Here we have velocity vectors but the idea is the same. To get to the new velocity vector we must ADD an additional bit of velocity as shown in green in Fig 3.5.

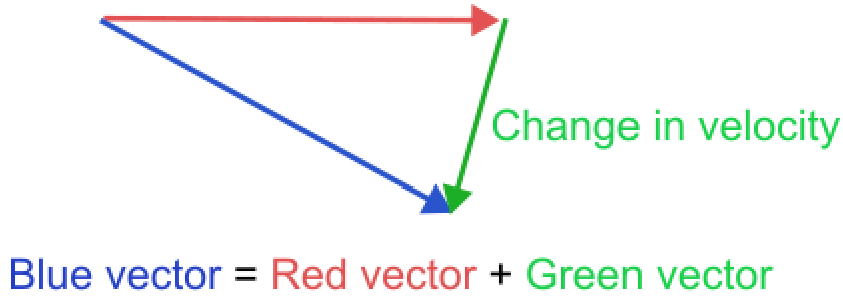

Fig 3.5

So we have added to the velocity, but we have not changed the speed. *Adding to a velocity means there must be an acceleration!* This *change in velocity* caused by the acceleration is shown in the last picture by the green arrow.

Now the crux of the matter. This extra bit of velocity, due to the acceleration, is directed TOWARDS THE CENTRE of the circle along the direction of the green arrow. You never get an acceleration without a force – Newton's second law from the last chapter says this. So, to get something to move in a circle there must be **a force towards the centre of that circle.**

Recap

1. Acceleration does not necessarily mean a change of speed.
   Acceleration is a vector it has direction as well as size. It is the rate at which the velocity is changing. In circular motion the velocity is continuously changing direction so the object moving in the circle IS accelerating.
2. There needs to be a force producing this acceleration and this force must be directed towards the centre of the circle. It is this force that prevents the object flying off at a tangent. There is NO OUTWARD "CENTRIFUGAL" FORCE. (This is an invention by mathematicians who prefer to reduce problems of dynamics to problems of statics – it is Ok for those who know what they are doing but really creates a muddle for those who don't).

*********BOB INTERRUPTS********

Bob: When I go round a corner in a car I am thrown outwards. Surely this is obvious. If it's not I want Alice to explain. You can put the italics on.

Alice: It is very easy to think what you say is true, but let's think about this a bit more carefully. Stand by a wall and lean against it. Hopefully the wall is stable and so you don't fall over. Clearly you are pushing against the wall. If you had a cushion between you and the wall it would be squashed. What is the force on you? This of our chat in the last chapter. Common sense will tell you it is into the room away from the wall. If it were not there you would fall over. So here we have Newton 3. You are pushing against the wall, and the wall is pushing you with and equal force in the opposite direction.

Bob: Yes, that's just like the explanation you gave before.

Alice: Now lean against the wall but close your eyes. Is this feeling any different from the force you might feel against with the door of the car as you go round a corner. It isn't. When you go round a corner fast and you are in contact with the door, the door is pushing you inwards towards the centre of the curve. It is preventing you flying off at a tangent. It is true that there is an equal and opposite force on the door which hopefully has a good catch. This is the same value as the inward force on you. Action of door on you pushing you inwards – reaction of you on the door pushing the door outwards.

Bob: I think I might be getting it.

Alice: Well, let's have another example. A string whirled around you head - maybe David with his sling about to slay Goliath. You think there is an outward force on the stone because there is an outward tug on your hand and yes there is such a force. But this force is on YOU. The force on the stone is inwards preventing the stone flying off at a tangent. The force inwards on the stone and the force outwards on your hand (connected to the stone with the string) are just equal and opposite pairs – the action and reaction in Newton 3. When you let go the stone will move along the tangent as there is no force keeping in the circle. If there were an outward force on the stone, it would fly directly outwards – which does not happen.

Circular motion is a case where experience can create totally false ideas.

Bob: I see. Without an inward force I would just keep going in the same direction along a straight line tangent. The inward force pushes me back onto the circular path.

Alice: Perfectly put. I don't think he has any more to say. But I better switch off the italics in case.

I sometimes wonder who is writing this!