4. Newton's Law of Gravity

Tycho Brahe (1546 1601) and Johannes Kepler (1571 – 1630)

You may ask why are we talking about Brahe and Kepler when the essay title is "Newton's Law of Gravity". It is because laws of planetary motion as proposed as a result of Brahe's and Kepler's work were vitally important when Newton developed his theory of Gravitation. They were explicable by Newton and his new theory and gave that theory immediate support. The measurements of Brahe and the mathematical genius of Kepler gave rise to **Kepler's three laws of planetary motion.**

Brahe was extremely precise in his measurements of positions of stars and planets. He constructed large elaborate instruments to measure positions with an accuracy previously unattainable, and all this was before the invention of the telescope.

Kepler was his assistant but had a mathematical ability that enabled him to process Brahe's observations and see how they fitted certain models of the planets, the sun and the moon. He was able from clever geometry to work out the relative distances of the planets from simply measurements of their positions and the times at those positions.

In 1609 he published his first two laws. It was ten years later in 1619 when he published the third law after the he had completed the difficult calculations on relative distances of the planets from the sun and their times for a complete orbit of the Sun. Here are the laws – we take them one at a time explaining each in turn.

Kepler's First Law

Planets orbit the sun in ellipses with the sun at one focus of the ellipse.

The simplest way to explain this is to describe how an ellipse can be drawn. An ellipse is a very specific geometrical shape – not just any oval. Take two drawing pins and pin them in a board a few centimetres apart. The make a loop with a piece of string and pass it over the drawing pins. If you then use a pencil with the string kept taut you will make an ellipse as the pencil moves as in Fig 4.1.

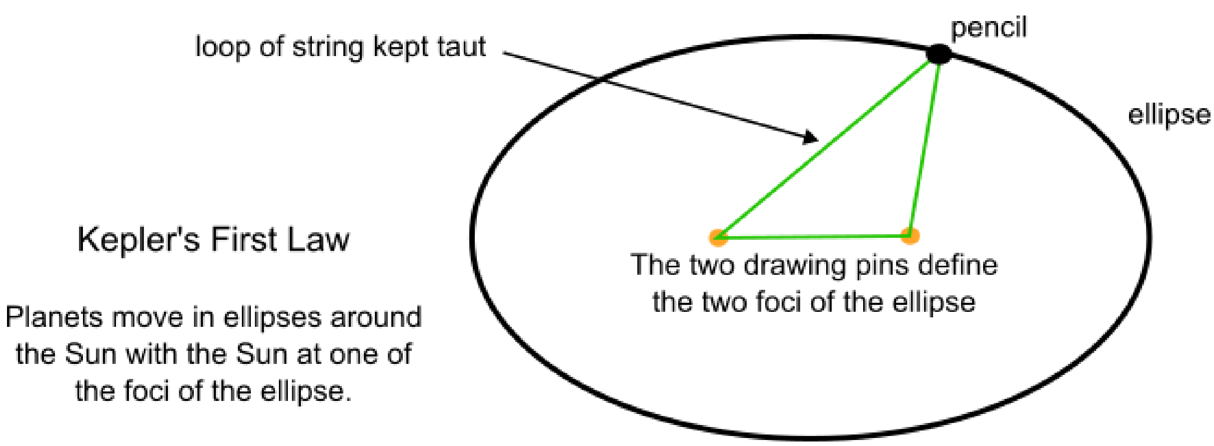

Fig 4.1

The two drawing pins are the two *foci* of the ellipse. The first law says that the path of any planet follows an ellipse and the sun is at one *focus* of the ellipse. This means that as the planet orbits, its distance from the sun varies. In the case of the Earth the two foci are relatively close. The closer the foci the nearer the orbit is to circular. The second focus for the Earth is about one thirtieth of the way from the Sun to the Earth. If you try this to scale with the drawing pins the shape you get is nearly a circle. The Earth is nearest to the Sun in January – about 147 million km and furthest in July - about 152 million km.

Kepler's Second Law

The speed of a planet in its orbit is such that the line joining the planet to the sun sweeps out equal areas in equal times.

This is best made clear by a picture. Fig 4.2 shows the position of a planet when it is near the sun together with its the position a set time later – points A and B. The area swept out by the line joining the planet to the Sun is show shaded in green. When the planet is far from the sun, the picture shows the planet's position and again the same set time later – points C and D. The area swept out is shown shaded in red. *Kepler's second law says that these two areas are equal*.

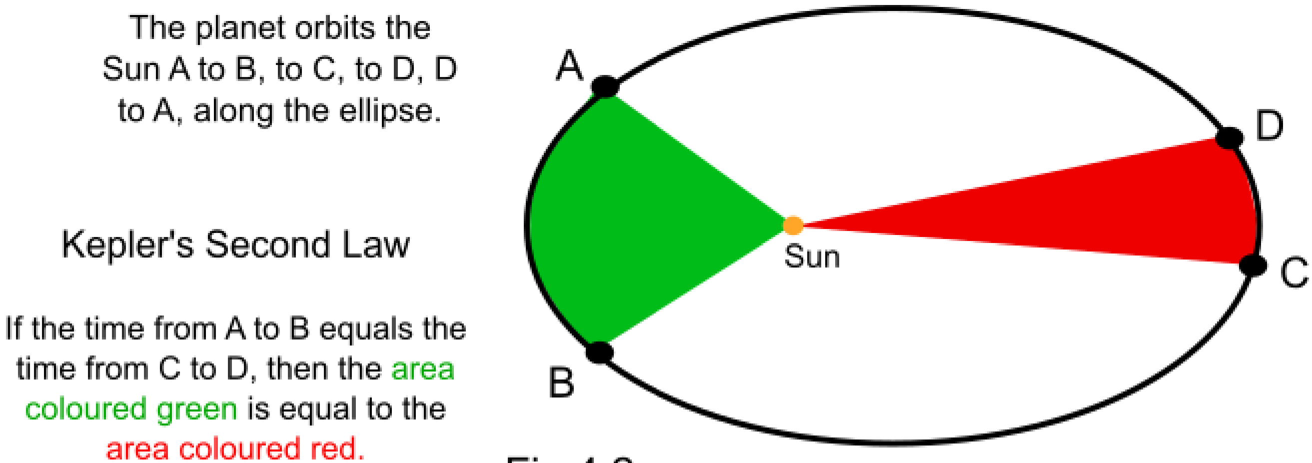

Fig 4.2

What this means is that when the planet is near the sun it must be moving faster than when the planet is far away - in order that the areas are the same.

Kepler’s Third Law

The square of a planet's orbital period is directly proportional to the cube of its average distance from the sun.

In the table that follows **T** is the time that the planet takes to go all the way round the Sun and **r** is the average distance from the Sun. The table below uses the results that Kepler derived from the measurements of Tycho Brahe. For each planet $\mathbf{r^2}$ - (r x r) and $\mathbf{T^3}$ (T x T x T) is worked out. Then the value of $\mathbf{r^2/T^3}$. This is entered in the last column.

Data used by Kepler (1618)

Planet	**Average distance from the sun compared to the Earth’s distance r**	**Period (days) T**	**r cubed divided by T squared** $\mathbf{r^2/T^3}$
Mercury	0.389	87.77	7.64
Venus	0.724	224.70	7.52
Earth	1	365.25	7.50
Mars	1.524	686.95	7.50
Jupiter	5.20	4332.62	7.49
Saturn	9.510	10759.2	7.43

Another way of saying that the period cubed T^3 is proportional to the average distance squared r^2 is that the ratio $\mathbf{r^2/T^3}$ is the same for all the planets. The last column shows this ratio and the values are very close. The measurements on which these figures are based were made before Brahe died in 1601 and without the use of telescopes. Modern measurements show that the values in the last column of the table are all very, very close to one another.

It is quite staggering that at this time it was possible to get position and time measurements to this degree of accuracy. Also staggering is Tycho’s ability to process these measurements to come up with his three laws which accurately describe how planets move and are still correct today.

******** BOB INTERRUPTS ***********

Bob: I want to talk to Alice.

Me: What is it now?

Bob: Nothing to mess up your “flow”, but you can switch italics on anyway.

Me: Very kind of you.

Alice: Hi Bob, what’s worrying you today?

Bob: Nothing really, I just can't get my head around how these guys years ago managed to work all this out. And what they found out is quite hard to get my head around.

Alice: There is a lot to take in, but the important point is to realize that these laws were known and they needed some explanation. Why ellipses? Why the change of speed as the planet gets nearer to the Sun, and why the peculiar rule that relates the time to go round the sun to the planet's distance from the Sun. This is what Newton managed. He put his laws of motion and his law of gravity together and lo-and-behold the mathematics predicted all of Kepler's laws. I think he will explain the law of gravity next.

Bob: I am still amazed at how Tycho could get measurements so exact and then for Kepler to do the sums to work out his laws.

Alice: I suppose it is because electricity had not been discovered and travel was by horse, that we think of their brain power as limited. There were very capable mathematicians and there were skilled workmen who could make the measuring equipment. Until the twentieth century a lot of science was done by wealthy folk who made their own equipment and did their own experiments. Science was at a stage where making discoveries did not need massive sums of money or Large Hadron Colliders.

Bob: Did anyone try any experiments with gravity?

Alice: Yes. In 1797–1798 by Lord Henry Cavendish actually showed two objects in his home (suitably modified) attracting one another. That's what comes next, I think. So, italics off and let's see

Newton's Law of Gravity.

We can think of Newton's law in two parts. The first part describes the very existence of gravity, and the second part gives the actual formula for calculating the value of the force of gravity.

The first part: Gravity is a force of attraction between **any** two objects. It is bigger if the masses of the objects are bigger. It is the word "any" here that may be the problem. We know that the Earth "has" in some sense gravity – but any two objects? The problem is that the force between ordinary sized objects is very, very small. But it can be shown to exist tolerably easily. Fig 4.3 shows a set up to do this. (You can do a search YouTube for this experiment or similar videos).

Fig 4.3

A long lath has heavy weights at each end and is suspended at its centre by a long fine thread just visible against the guy's dark clothes. The weights at the end are suspended *above the ground* so the lath can turn about the thread. Because the thread is thin and long the tiniest force will set the lath turning. After a long time waiting for the lath to stop moving two large bowling balls are then positioned as in Fig 4.4. They are resting on the floor so they cannot move.

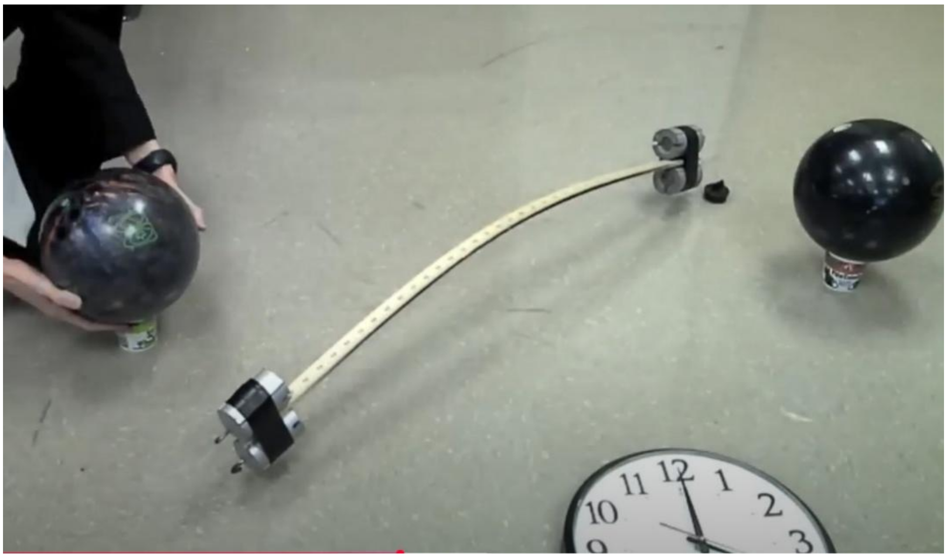

Fig 4.4

I you watch the video it takes a time lapse film and shows the clock going rapidly. There is a gravitational force of attraction between each bowling ball and the nearer weight on the lath. This force causes the lath to slowly twist into the position shown in Fig 4.5. The force is very small and the time to move is long, hence the reason for the time lapse images.

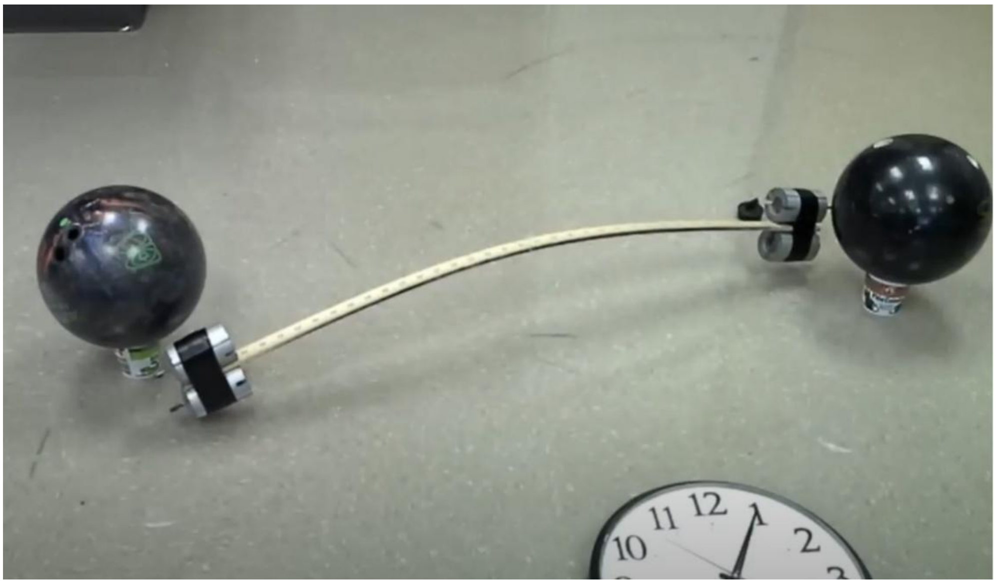

Fig 4.5

This shows that gravity does exert a force between any pair of objects, but we only normally notice it if large masses are involved. It took this unusual set-up and a long wait to show the extremely small force. It is true that the bowling balls attract one another but, unlike the weights on the end of the lath, they cannot easily move. It is also true that the bowling balls attract the weights at the far end of the lath. Gravity, however, gets weaker with distance so the effect of this is much smaller than on the nearer weight.

Gravity has nothing to do with magnetism. Gravity is a universal force in its own right. If you ask what causes it then there is no answer. Even Einstein's theory of gravity - which is very different – simply moves the goalposts and just poses a different question. Masses attract one another according to the laws of gravity and that's it. That is what Newton said, although he *was* worried that such forces can "act at a distance".

Second part: So a force of attraction exists between any two objects. Newton's law actually says how big this force should be. It is easiest to simply state it as a formula and then explain what the formula means. In the formula F is the force of attraction (in newtons), m_1 and m_2 are the two attracting masses (in kg) and r (in metres) is the distance between their centres. *G is the Universal Constant of Gravitation* believed to be the same throughout the universe. The value of G is such that, unless one or both of the masses is very big, the force F is very small.

$$F = G\frac{m_1 m_2}{r^2}$$

We have talked about Newton's three laws of dynamics and now about his universal law of gravitation. The really key thing is:

When Newton put all this together he was able prove mathematically that Kepler's Laws are correct. The theory explains why planets move in ellipses the way they do. It explains why there is the peculiar relation between the time of orbit and average distance. It unifies these disparate facts. It is also true that these laws are used today in calculating the motion of spaceships and in working out the times and duration of the engine burns. The engineers helping to rescue Apollo 13 would have based their calculations on Newton's laws (albeit with computer assistance for the sums).

Measuring G.

To get actual answers for the force we can easily measure the masses and distance apart but we also need the value of G. This was first measured in 1797–1798 by Henry Cavendish. His experimental set up was similar to the one in the pictures above with lead spheres each 0.73 kg on the end of a rod suspended by a very fine thread. He brought up two massive 158 kg lead balls positioned 22.5 cm from the suspended spheres. Using the angle that the thread was twisted against the resistance of the thread to twist, he was able to work out the force. He knew the masses and their separation so he could then calculate G.

Modern methods have improved on Cavendish, but the force is very small and so it remains difficult to measure. The currently accepted value for G is $6.693(34)\times10^{-11}$ ($m^3kg^{-1}s^{-2}$ don't worry about the units) and is still subject to a 0.1% uncertainty. This is quite a high degree of uncertainty for one of the fundamental constants of nature. Most are known to much greater degree of accuracy.